(1) No books, notes or calculators are allowed.
(2) Show your work in details.

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(1) (30 points) Which of the following statements are true and which are false? Justify your answer.

(a) The set \( \{(a, b, c) : a + b = c\} \) is a subspace of the vector space \( \mathbb{R}^3 \).

(b) If \( A^2 = A \), then \( (I - A)^2 = I - A \), where \( A \) is a square matrix.
(c) If the characteristic polynomial of an $n \times n$ matrix $A$ is $\lambda^n + 1$, then $A$ is invertible.

(d) Every $3 \times 3$ skew symmetric matrix is singular.
(e) If \( u \) cannot be expressed as a linear combination of \( v \) and \( w \), then the three vectors \( u, v \) and \( w \) are linearly independent.

\[
(A + B)(A - B) = A^2 - B^2
\]
for all square matrices \( A \) and \( B \) of the same size.
(2) (20 points) Solve the system of linear equations

\[
\begin{align*}
2x_1 + 2x_2 + 2x_3 &= 0 \\
-2x_1 + 5x_2 + 2x_3 &= 1 \\
8x_1 + x_2 + 4x_3 &= -1
\end{align*}
\]
(3) (25 points) For which real values of \( \lambda \) do the following vectors
\[
\mathbf{v}_1 = (1, 2, \lambda), \quad \mathbf{v}_2 = (1, 4, \lambda^2), \quad \mathbf{v}_3 = (1, 8, \lambda^3)
\]
form a linearly dependent set in \( \mathbb{R}^3 \)?
(4) (25 points) Find the inverse, determinant, characteristic polynomial, eigenvalues and eigenvectors of the matrix

\[
\begin{bmatrix}
1 & 2 & 3 \\
0 & 2 & 3 \\
0 & 0 & 3 \\
\end{bmatrix}
\]