Math 214 Midterm Review[1]

Sections covered: 11.1, 11.2, 11.3, 11.4, 11.5, 11.6, 11.7, 11.8, 11.9, 11.10

Some sample problems:

(1) For each series below, determine whether there is absolute convergence, conditional convergence, or divergence. Give reasons for your answers. You may assume the knowledge of the basic convergence and divergence theorems in class, and that of the geometric series, and the divergence of the harmonic series $\sum_{n=1}^{\infty} 1/n$. You must justify your answer.

(a) $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n}{n}$

(b) $\displaystyle\sum_{n=2}^{\infty} \left(1 + \frac{(-1)^n}{\ln(n)}\right)$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{n^n}$

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{3^n n}{5^n(n+1)}$

(2) Write down an expression to estimate the integral

$$\int_0^{0.2} \cos(x^2)dx$$

which estimates L within an error < 0.001. Justify your answer. (You need not simplify your answer.)

(3) Let $f(x) = \cos(x) + \sin(x)$. Compute the first three terms (2nd Taylor polynomial $T_2(x) = f_2(x)$) of the Taylor series for $f(x)$ about the point $x = \pi$.

(4) For each function below, find its Taylor series and its radius of convergence.

(a)

$$f(x) = \frac{x^2}{(x+1)(x+2)}$$

about $x = 3$.

[1] http://www.math.ualberta.ca/~xichen/math21409w/p1.pdf

(b)
$$f(x) = 2^x + 3^x$$
about $x = 1$.

(5) (a) For what values of x does the series $\sum_{n=1}^{\infty} \frac{(-3)^n x^n}{2^n}$ converge/diverge?

(b) Show that
$$-\frac{x}{2(1+x/2)^2} = \sum_{n=1}^{\infty} \frac{(-1)^n n x^n}{2^n}$$
for $-2 < x < 2$.

(6) Evaluate
$$\sum_{n=0}^{\infty} \frac{1}{(n+1)(n+4)}$$