

PRINT NAME: \_\_\_\_\_

STUDENT ID NUMBER: \_\_\_\_\_

- (1) No books, notes or calculators are allowed.
- (2) Show your work in details.

Problem	Points	Score
1	20	
2	20	
3	20	
4	20	
5	20	
Total	100	

- (1) (20 points) For each series below, determine whether there is absolute convergence, conditional convergence, or divergence. Give reasons for your answers. You may assume the knowledge of the basic convergence and divergence theorems in class, and that of the geometric series, and the divergence of the harmonic series  $\sum_{n=1}^{\infty} 1/n$ . You must justify your answer.

(a) 
$$\sum_{n=2}^{\infty} \frac{2^n}{3^n + 5}$$

(b) 
$$\sum_{n=2}^{\infty} (1 + \cos(n\pi))$$

$$(c) \sum_{n=1}^{\infty} \frac{(-1)^n}{n!}$$

$$(d) \sum_{n=1}^{\infty} \frac{n^{10}}{10^n}$$

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- (2) (20 points) Write down an expression to approximate the integral

$$\int_0^{0.1} x^2 e^{-x^2} dx$$

with an error of magnitude  $< 10^{-5}$ . You need not simplify your answer but must justify it.

- (3) (20 points) Let  $f(x) = \tan x$ . Compute the first three terms (2nd Taylor polynomial  $T_2(x) = f_2(x)$ ) of the Taylor series for  $f(x)$  about the point  $x = \pi/4$ .

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(4) (20 points) Find the Taylor series of the function

$$f(x) = 2^x + 3^x$$

about the point  $x = 1$  and its radius of convergence.

(5) (20 points) For what values of  $x$  does the series  $\sum_{n=2}^{\infty} \frac{x^n}{\ln n}$  converge/diverge? Justify your answer.