Math 214 Final Review[1]

Sections covered: 11.1, 11.2, 11.3, 11.4, 11.5, 11.6, 11.7, 11.8, 11.9, 11.10, 10.4, 10.5, 10.7, 12.1, 12.2, 12.3, 12.4, 12.5, 13.1, 13.3, 13.4, 13.5, 14.1, 14.2, 14.3, 14.4, 14.5, 14.6, 14.7, 14.8

Some sample problems:

(1) Find the point of intersection of the lines $x = t + 2, y = 3t + 4, z = 4t + 5$, and $x = 6s + 13, y = 5s + 11, z = 4s + 9$, and then find the plane containing these two lines.

(2) Let $w = f(u, v)$, $u = x + y$, and $v = xy$. Find $\partial w/\partial x$ and $\partial w/\partial y$ and express your answers in terms of x, y, $\partial w/\partial u$ and $\partial w/\partial v$.

(3) Let
$$f(x, y) = (4x - x^2)\cos y$$
  (a) Find all the critical points of $f(x, y)$ and identify local maxima, local minima and saddle points among them.
  (b) Find the absolute maximum and minimum of $f(x, y)$ in the region given by $1 \le x \le 3$ and $-\pi/4 \le y \le \pi/4$.

(4) Find an equation for the plane through points $P = (-1, 1, 0)$, $Q = (8, -3, -1)$ and $R = (-4, 1, 1)$ and the area of the triangle ΔPQR.

(5) Let
$$f(x) = \frac{1}{(x + 1)(x + 2)(x + 3)}$$
  (a) Find the Taylor series of $f(x)$ about the point $x = 0$ and its radius of convergence.
  (b) Compute the value of
$$\sum_{n=0}^{\infty} f(n)$$

(6) Let
$$f(x, y, z) = x^3 + x^2 y + y^2 z$$
  (a) Find the gradient ∇f of f.
  (b) Suppose that the surface $f(x, y, z) = c$ passes through the point $(1, 2, 1)$. Find the constant c and the equation of the tangent plane to the surface at $(1, 2, 1)$.

(7) (a) Maximize $f(x, y, z) = xyz$ subject to the constraints $x^2 + y^2 + z^2 = 1$.

[1] http://www.math.ualberta.ca/~xichen/math21409w/fp.pdf

(b) Find the dimensions of the rectangular box with the largest volume that can be inscribed in the unit sphere.

(8) Let C be a plane curve given by

$$\mathbf{s}(t) = \frac{t^3}{3}\mathbf{i} + \frac{t^2}{2}\mathbf{j}$$

(a) Find the length of C for $1 \leq t \leq 2$.

(b) Find the curvature of C at $t = 1$.

(c) Find the osculating circle to the curve C at $t = 1$.