XI CHEN	Math 214 Final	Apr 22, 2009

PRINT NAME:______________________________

STUDENT ID NUMBER:______________________________

(1) No books, notes or calculators are allowed.
(2) Show your work in details.

Problem	Score	Problem	Score
1 (30)		2 (20)	
3 (20)		4 (30)	
5 (20)		6 (25)	
7 (25)		8 (30)	
Total (200)			

(1) (30 pts) Let

$$f(x) = \cos^2(x^2)$$

(a) (15 pts) Find the Taylor series of $f(x)$ about the point $x = 0$ and its radius of convergence.

(b) (15 pts) Write down an expression to approximate the integral

$$\int_0^{0.1} f(x)dx$$

with an error of magnitude $< 10^{-5}$. You need not simplify your answer but must justify it.

(2) (20 pts) Find an equation for the plane through points $P = (-1, 1, 0)$, $Q = (8, -3, -1)$ and $R = (-4, 1, 1)$ and the area of the triangle ΔPQR.

(3) (20 pts) Let $w = f(x, y)$, $x = r\cos\theta$, and $y = r\sin\theta$. Find $\partial w/\partial r$ and $\partial w/\partial\theta$ and express your answers in terms of r, θ, f_x and f_y.

(4) (30 pts) Let

$$f(x, y) = x^4 + y^4 + 4xy$$

(a) (15 pts) Find all the critical points of $f(x, y)$ and identify local maxima, local minima and saddle points among them.

(b) (15 pts) Find the absolute maximum and minimum of $f(x, y)$ in the region given by $-2 \leq x \leq 2$ and $-2 \leq y \leq 2$.

(5) (20 pts) Find an equation of the plane through the point $(2, 1, -1)$ and perpendicular to the line of intersection of the planes $2x + y - z = 3$ and $x + 2y + z = 2$.

(6) (25 pts) Find the points on the surface

$$xy + yz + zx - x - z^2 = 0$$

where the tangent plane is parallel to the xy-plane.

(7) (25 pts) Find the point closest to the origin on the curve of intersection of the plane $2y+4z = 5$ and the cone $z^2 = 4x^2+4y^2$.

(8) (30 pts) Let C be a plane curve given by

$$\mathbf{s}(t) = (2\ln t)\mathbf{i} - \left(t + \frac{1}{t}\right)\mathbf{j}$$

(a) (10 pts) Find the length of C for $1 \leq t \leq 2$.

(b) (10 pts) Find the curvature of C at $t = 1$.

(c) (10 pts) Find the osculating circle to the curve C at $t = 1$.