Math 214 Final Review


Some sample problems:

1. Find an equation for the plane through points \( P = (1, -1, 2), \) \( Q = (2, 1, 3) \) and \( R = (-1, 2, -1) \) and the area of the triangle \( \Delta PQR. \)

2. Let \( f(x) = \frac{1}{(x + 1)(x + 4)} \)
   (a) Find the Taylor series of \( f(x) \) about the point \( x = 0 \) and its radius of convergence.
   (b) Do the same for \( f(x) \) about the point \( x = 1. \)
   (c) Compute the value of \( \sum_{n=1}^{\infty} f(n) \)

3. Find the point of intersection of the lines \( x = t, y = -t + 2, z = t + 1, \) and \( x = 2s + 2, y = s + 3, z = 5s + 6, \) and then find the plane containing these two lines.

4. Let \( f(x, y, z) = xy + yz + zx \)
   (a) Find the gradient \( \nabla f \) of \( f. \)
   (b) Find the equation of the tangent plane to the surface \( f(x, y, z) = 5 \)
      at \( (1, 1, 2). \)
   (c) Let \( g(t) \) be a function that is differentiable at \( t = 5 \) with \( g(5) = g'(5) = 1 \) and let \( F(x, y, z) = g(f(x, y, z)). \) Find \( \partial F/\partial x, \partial F/\partial y \) and \( \partial F/\partial z \) at \( (1, 1, 2). \)

5. Let \( C \) be a plane curve given by \( s(t) = (t \sin t + \cos t)i + (t \cos t - \sin t)j \)
   (a) Find the length of \( C \) for \( 0 \leq t \leq \pi/2. \)
   (b) Find the curvature of \( C \) at \( t = \pi/2. \)
   (c) Find the osculating circle to the curve \( C \) at \( t = \pi/2. \)

http://www.math.ualberta.ca/~xichen/math21408f/fp.pdf
(6) Let 
\[ f(x, y) = 4xy - x^4 - y^4 + 16 \]
(a) Find all the critical points of \( f(x, y) \) and identify local maxima, local minima and saddle points among them.
(b) Find the absolute maximum and minimum of \( f(x, y) \) in the triangular region bounded by the lines \( y = -2 \), \( y = x \) and \( x = 2 \).

(7) Let \( w = f(r, \theta) \), \( r = \sqrt{x^2 + y^2} \), and \( \theta = \tan^{-1}(y/x) \). Find \( \frac{\partial w}{\partial x} \) and \( \frac{\partial w}{\partial y} \) and express your answers in terms of \( r \) and \( \theta \).

(8) Determine the distance from the line \( y = x + 1 \) to the parabola \( y^2 = x \).

(9) Minimize \( f(x, y, z) = xyz \) subject to the constraints \( x^2 + y^2 - 1 = 0 \) and \( x - z = 0 \).