MATHEMATICS 113 (A1)
Final Examination

Fall 2010

Date: Monday, December 13, 2010

Time: 2 Hours

LAST NAME: ___________________ FIRST NAME: ___________________
(Please, print)

I.D.: __________________________________________

Instructions

1. Books, notes or calculators are not permitted.
2. Show all your work.
3. Make sure your examination paper has 6 questions.

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1. (20 points)

(a) Find an equation of the tangent line to the curve 
\[ x^3 + y^3 - 9xy = 0 \]
at the point \( P(2, 4) \).
Answer: \( y = \frac{4}{5}x + 12/5 \)

(b) If \( f''(t) = t^3 + 4t - 5 \) and \( f'(0) = 1, f(0) = -2 \) find \( f(t) \).
Answer: \( f(t) = \frac{t^5}{50} + \frac{2t^3}{3} - \frac{5t^2}{2} + t - 2 \)

(c) Find \( g'(-\frac{\pi}{4}) \) if \( g(x) = \cos x + \int_{\tan x}^{1} \sin(1 + t^3)dt \).
Answer: \( g'(-\frac{\pi}{4}) = 1/\sqrt{2} \)

(d) Find the critical numbers of \( f(x) = x^4(x - 9) \).
\( x = 0, x = 4 \)

(e) Find the absolute maximum and absolute minimum values of the function \( f(x) = 3x^5 - 5x^3 \) on the interval \([-2, 2]\).
Answer: Abs.Max. \( f(2) = 56 \) Abs.min. \( f(-2) = -56 \)

2. (20 points) The function \( f \) is defined by \( f(x) = \frac{x(x - 3)}{(x + 3)^2} \). Its first and second derivatives are
\[ f'(x) = \frac{9x - 9}{(x + 3)^3}, \quad f''(x) = \frac{-18x + 54}{(x + 3)^4}. \]
Find each of the following:

(a) The domain of \( f \) and intercepts with \( x \) and \( y \) axes.
(b) The intervals where \( f \) is increasing and where \( f \) is decreasing.
(c) Local extreme values of \( f \), if any.
(d) Intervals where \( f \) is concave upward and concave downward and inflection points.
(e) All asymptotes
(f) Sketch the graph of \( f \).

Answers: (a) \((-\infty, -3) \cup (-3, \infty), (0, 0), (3, 0)\)
(b) \( f \) increases on \((-\infty, -3) \cup (1, \infty)\) \( f \) decreases on \((-3, 1)\)
(c) local min \( f(1) = -1/8 \).
(d) \( f \) concave upward on \((-\infty, -3) \cup (-3, 3), f \) concave downward on \((3, \infty)\); inflection point \( (3, 0) \)
(e) \( x = -3 \) V.A. \( y = 1 \) H.A.
3. (10 points) The function $f$ is defined by $f(x) = \sqrt{x+3}$.
   (a) Find the linearization of $f$ at $x = 0$.
       Answer: $L(x) = \sqrt{3} + \frac{1}{2\sqrt{3}}x$
   (b) Find the point(s) on the graph of $f$ where the tangent line is parallel to the secant line through $(-3,0)$ and $(6,3)$.
       Answer: $P(-3/4,3/2)$

4. (20 points) Evaluate the following integrals:
   (a) $\int \frac{1}{x^2} \sin\left(\frac{1}{x}\right) \cos\left(\frac{1}{x}\right) dx$,
   (b) $\int \frac{1}{(2x+3)^2} dx$,
   (c) $\int_0^1 18x^3(3x^2 + 1)^{\frac{1}{2}} dx$,
   (d) $\int_0^{\pi/2} |\sin x| dx$.

       Answer: (a) $-\frac{\sin^2(\frac{1}{x})}{2} + C$
   (b) $-\frac{1}{2(2x+3)} + C$
   (c) $\frac{116}{15}$
   (d) $3 - \frac{1}{\sqrt{2}}$

5. (10 points) Suppose a camera is filming the launch of a rocket from 10km away from a launching pad. The rocket is traveling vertically. At certain moment, the angle between the camera and the ground is equal $\pi/3$ and is changing at the rate $0.5 rad/min$. What is the rocket’s velocity at that moment?
       Answer: $20 km/min$

6. (20 points)
   (a) Use the definition of the definite integral as the limit of the Riemann sum to evaluate
   $\int_0^3 [4 - (x - 2)^2] dx$.

       Hint: $\sum_{i=1}^{n} i = \frac{n(n + 1)}{2}$, $\sum_{i=1}^{n} i^2 = \frac{n(n + 1)(2n + 1)}{6}$. 
(b) Evaluate the integral \( \int_{0}^{3} [4 - (x - 2)^2] dx \) using Part 2 of the Fundamental Theorem of Calculus.

(c) Draw a diagram to explain the geometric meaning of the integral in part (a).

Answer: 9