1. (20 points)

(a) Find the domain of $f(x) = \sqrt{\frac{5-x}{x-2}}$.

Answer: $(2, 5]$  

(b) Find an equation of the tangent line to the curve $y = \sqrt{x^4 + 3}$ at the point $(-1, 2)$.

Answer: $y = -x + 1$

(c) Evaluate $\lim_{x \to 0} \frac{2x}{\tan(3x)}$.

Answer: $\frac{2}{3}$

(d) Let $f$ be a continuous function on $[a, b]$. Which of the following statements is FALSE:

(A) $f$ always has an absolute maximum on $[a, b]$.

(B) $f$ always has an absolute minimum on $[a, b]$.

(C) If $f'(c) = 0$ for some $c$ in $(a, b)$, then $f$ has a local extremum at $c$.

(D) If $f$ has a local extremum $x = c$ in $(a, b)$, then $c$ is a critical number.

Answer: C

(e) Find the horizontal asymptotes of the function $f(x) = \frac{1-2x}{\sqrt{1+x^2}}$.

Answer: $y = -2$ at $\infty$ and $y = 2$ at $-\infty$

(f) If $f(x) = x \ln(2x^2)$, find $f'(1)$.

Answer: $\ln 2 + 2$

(g) If $g(x) = 2x - \int_0^{\tan x} \frac{1}{\sqrt{1+t^2}} dt$, $x \in (0, \frac{\pi}{2})$ find $g'(\frac{\pi}{3})$.

Answer: 0

(h) If $g(x) = [f(x)]^{\frac{1}{3}}$ and $f(1) = 8$, $f'(1) = 3$ find $g'(1)$.

Answer: $\frac{1}{4}$

(i) Evaluate the integral

$$\int_0^3 \left( 2 + \sqrt{9 - x^2} \right) dx$$

by interpreting the integral in terms of area:

Answer: $6 + \frac{9}{4}$

(j) Evaluate $\int_0^{\frac{\pi}{2}} \cos(2x)dx$.

Answer: $\frac{1}{2}$
2. (20 points) Given \( f(x) = \frac{x^2 + x - 1}{x^2} \), \( f'(x) = \frac{2 - x}{x^3} \), \( f''(x) = \frac{2(x - 3)}{x^4} \), find each of the following:

(a) The domain of \( f \) and intercepts with \( x \) and \( y \) axes.
(b) The intervals of increase or decrease.
(c) The local maximum and minimum values.
(d) The intervals of concavity and inflection points.
(e) Vertical and horizontal asymptotes.
(f) Sketch the graph of \( f \).

**ANSWERS:**

(a) \( D_f = (-\infty, 0) \cup (0, +\infty) \)

(b) \( f \) increases on \((0, 2)\); \( f \) decreases on \((-\infty, 0) \cup (2, +\infty) \)

(c) \( f \) has no local minimum; Local max. at \( x = 2, \ f(2) = 5/4 \)

(d) \( f \) is concave upward on \((3, \infty)\)

(e) H.A, \( y = 1 \) V.A. \( x = 0 \)

3. (10 points) A small funnel in the shape of a cone is being emptied of fluid at the rate of 12 cubic centimeters per second (the tip of the cone is downward). The height of the cone is 8 cm and the radius of the top is 4 cm. How fast is the fluid level dropping when the level stands 2 cm above the vertex of the cone?

**HINT:** The volume of a cone is \( V = \frac{1}{3}\pi r^2 h \).

Answer: fluid level is dropping at \( \frac{12}{\pi} \) cm/s
4. (10 points) A farmer plans to fence a rectangular pasture adjacent to a river. The pasture must contain 180,000 square meters in order to provide enough grass for the herd. What dimensions would require the least amount of fencing if no fencing is needed along the river?

Answer: 600 × 300

5. (10 points)
   
   (a) Find $f(x)$ if $f''(x) = \frac{1+\cos(2x)}{2}$, $f'(\frac{\pi}{2}) = 1$ and $f(\frac{\pi}{2}) = 0$.
   
   (b) Find the absolute maximum and absolute minimum values of $f(t) = 2 \cos t + t$ on $[0, \frac{\pi}{2}]$.

Answer:

(a) $f(x) = \left(\frac{1}{4}\right)x^2 - \left(\frac{1}{8}\right)\cos(2x) + x - \left(\frac{\pi}{4}\right)x + \frac{\pi^2}{16} - \pi/2 - 1/8$

(b) abs max: $f(\pi/6) = \sqrt{3} + \pi/6$. abs.min. $f(\pi/2) = \pi/2$.

6. (10 points) Evaluate the following integrals:

   (a) $\int_4^0 e^{\sqrt{x}} dx$, 
   
   (b) $\int \frac{x}{3 - 2x^2} dx$.

Answer:

1. $\int_4^0 e^{\sqrt{x}} dx = 2(e^2 - 1)$

2. $\int \frac{x}{3 - 2x^2} dx = \left(-1/4\right) \ln |3 - 2x^2| + C$.

7. (20 points) Evaluate the integral $\int_0^1 (3x^2 + 2x) dx$ in two ways:

   (a) Using the definition of the definite integral

   Hint: $\sum_{i=1}^{n} i = \frac{n(n + 1)}{2}, \sum_{i=1}^{n} i^2 = \frac{n(n + 1)(2n + 1)}{6}$.

   No marks will be given if the definition is not used.

   (b) Using Part 2 of the Fundamental Theorem of Calculus.

Answer: 2