**Instructions**

1. This is a closed book exam. *No books, notes, calculators or other electronic aids are allowed!*

2. There is one *multiple choice question* with ten parts and six *long answer questions*; for the long answer questions you must show all your work.

3. *A mark of zero will be given for multiple choice questions with more than one circled answer.*

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1. (30 points) Circle your answer

(a) The domain of \( f(x) = \ln(25 - x^2) \) is:
   - (A) \((-\infty, -5) \cup (-5, 5) \cup (5, \infty)\)
   - (B) \((-\infty, -5) \cup (5, \infty)\)
   - (C) \((-5, 5)\)
   - (D) \([-5, 5]\)

(b) The slope of the tangent line to the curve \( \sin(x - y) = 2x + 2y \) at the point \((\pi, -\pi)\) is:
   - (A) \(-1/3\)
   - (B) \(2/3\)
   - (C) \(1/3\)
   - (D) None of the above

(c) \[ \lim_{x \to 2^+} \frac{x^2 - 3x + 2}{|2 - x|} = \]
   - (A) 1
   - (B) -1
   - (C) does not exist
   - (D) None of the above

(d) If \( h(x) = 2f(x) \), where \( f(1) = 2 \) and \( f'(1) = 3 \), then \( h'(1) \) equals:
   - (A) 4
   - (B) \(\ln(2^{12})\)
   - (C) 12
   - (D) None of the above
(e) The vertical asymptotes of the function \( f(x) = \frac{x^2 - 25}{x^2 - 6x + 5} \) are:

(A) \( x = 1 \) and \( x = 5 \)  (B) \( x = 1 \)

(C) \( x = 5 \)  (D) None of the above

(f) The absolute maximum value of \( f(x) = x^2 + 2x + 2 \) on \([-4, 1]\) is:

(A) 1  (B) 5

(C) 10  (D) None of the above

(g) If \( g(x) = \int_{\cos x}^{0} \frac{1}{\sqrt{1 + t^2}} \, dt \) then \( g'(\frac{\pi}{2}) \) equals:

(A) 1  (B) \(-1\)

(C) 0  (D) \( \pi \)
(h) The limit \( \lim_{n \to \infty} \sum_{i=1}^{n} \left[ 1 - \left( 1 + \frac{3i}{n} \right)^2 \right] \frac{3}{n} \) as a definite integral is

(A) \( \int_{0}^{\frac{3}{4}} (1 - (1 + x)^2) \, dx \)

(B) \( \int_{1}^{4} (1 - (1 + x)^2) \, dx \)

(C) \( \int_{0}^{\frac{3}{4}} (1 - x^2) \, dx \)

(D) None of the above

(i) If \( f'(x) = 2e^x + 3\sin(\pi x) \) and \( f(0) = f(1) = 0 \), then \( f(x) = \)

(A) \( 2e^x + 3\sin(\pi x) + (2 - 2e)x - 2 \)

(B) \( 2e^x - \frac{3}{\pi^2} \sin(\pi x) + (2 - 2e)x - 2 \)

(C) \( 2e^x - 3\sin(\pi x) + (2 - 2e)x - 2 \)

(D) None of the above

(j) \( \int_{0}^{1} (x^2 + \pi^2) \, dx = \)

(A) \( \frac{\pi^2}{\pi + 1} \)

(B) \( \frac{1}{\pi + 1} + \frac{\pi - 1}{\ln \pi} \)

(C) \( \frac{1}{\pi + 1} + (\pi - 1) \ln \pi \)

(D) None of the above
2. (15 points) Evaluate the integral \( \int_1^3 (1 - 3x^2) \, dx \) using the definition of the definite integral. Use the right-endpoints.

Hint: \( \sum_{i=1}^{n} i = \frac{n(n + 1)}{2}, \sum_{i=1}^{n} i^2 = \frac{n(n + 1)(2n + 1)}{6}. \)

No marks will be given if the definition is not used.
3. (10 points) A man of 6 ft tall walks toward a wall at a speed of 4 ft/s. A light is located on the ground 60 ft from the wall and casts a shadow of the man onto the wall (assuming the light lies on the path of the man). At what rate is the length of the shadow changing when the man is 20 ft from the wall?
4. (15 points) A box with a square bottom, vertical sides and no top is to be built so as to contain 16 $m^3$ of liquid. If the cost of the bottom material is $2$ per square meter and the cost of the side material is $1$ per square meter, find the dimensions of the box which will give the lowest cost.
5. (10 points) Evaluate the following integrals:

(a) \( \int_0^5 \frac{|x^2 - 9|}{x + 3} \, dx, \)

(b) \( \int \frac{\cos(4 \ln x)}{3x} \, dx. \)
6. (20 points)

Let \( f(x) = \frac{2(x + 2)(x - 1)}{x^2} \). Given that \( f'(x) = \frac{8 - 2x}{x^3} \) and \( f''(x) = \frac{4(x - 6)}{x^4} \),

find each of the following:

(a) The domain of \( f \) and intercepts with \( x \) and \( y \) axes.

(b) The intervals of increase or decrease.

(c) The local maximum and minimum points.

(d) The intervals of concavity and inflection points.

(e) Vertical and horizontal asymptotes.

(f) Sketch the graph of \( f \). Label all asymptotes, intercepts, local max and min points and inflection points.