Math 113, Fall 2008, Final Examination

(1)
(a) If \( f(2) = 2 \) and \( f'(2) = -3 \), find \( \frac{d}{dx}\left[\frac{f(x)}{(1+x^3)^{1/2}}\right] \) at \( x = 2 \). Answ: -13/9
(b) Evaluate \( \lim_{x \to -\infty} \frac{2x-1}{\sqrt{3x^2+2}} \). Answ: \(-2/\sqrt{3}\)
(c) If \( f'(t) = \sqrt{t}(6+5t) \) and \( f(10) = 10 \), find \( f(t) \). Answ: \( f(t) = 4t^{3/2} + 2t^{5/2} + 10 - 240\sqrt{10} \)
(d) Find point(s) on the curve \( y = \sqrt{x-1} \) where the tangent line is parallel to the line passing through \( A(1,0) \) and \( B(3,\sqrt{2}) \). Answ: \((3/2,1/\sqrt{2})\).
(e) If \( f(x) = \int_{\cos(x^2)}^{5} (1+t)dt \), find \( F'(\sqrt{\pi}/2) \). Answ: \(-\sqrt{2\pi}\).

(2) Consider the curve given by the equation
\[
x^2 - \sqrt{3}xy + 2y^2 = 5.
\]
(a) Find two points on the curve where \( x = \sqrt{3} \). Answ: \((\sqrt{3},2)\), \((\sqrt{3},-1/2)\).
(b) Find equations of the tangent lines to the curve at each of the points obtained in part (a). Answ: \( y = 2 \), \( y = (\sqrt{3}/2)x - 2 \).

(3) Given \( f(x) = 2x - 3x^{2/3} \), \( f'(x) = 2 - 2x^{-1/3} \), \( f''(x) = \frac{2}{3}x^{-4/3} \), find each of the following:
(a) The domain of \( f \)
(b) \( x- \)intercepts and \( y- \)intercepts
(c) Intervals where \( f \) is increasing/decreasing
(d) Critical points, local max/min values of \( f \)
(e) Intervals where \( f \) is concave up/down and inflection points
(f) Sketch the graph
Answers:
(a) \( (-\infty, \infty) \)
(b) \((0,0)\), \((27/8,0)\), \( x- \)intercepts; \((0,0)\) \( y- \)intercept
(c) \( f \) increases for \( x \in (-\infty, 0) \cup (1, \infty) \), \( f \) decreases for \( x \in (0,1) \)
(d) Critical numbers are $x = 0, x = 1$; $f(0) = 0$ is local max, $f(1) = -1$ is a local min

(e) $f$ is concave up for $x \in (-\infty, 0) \cup (0, \infty)$; no inflection point

(f) 

(4) Evaluate the following integrals:

(a) $\int \frac{1}{\sqrt{x(1 + \sqrt{x})}} \, dx$,

(b) $\int x^{\frac{1}{3}} \sin(x^{\frac{4}{3}} - 8) \, dx$,

(c) $\int_0^\pi (1 - \cos(3t)) \sin(3t) \, dt$,

(d) $\int 2x^2 \sqrt{2 - 3x} \, dx$.

Answers:

(a) $\frac{-2}{1 + \sqrt{2}} + C$

(b) $\frac{-3}{4} \cos(x^{\frac{4}{3}} - 8) + C$

(c) $1/2$

(d) $\frac{2}{27} \left[ \frac{8}{13}(2 - 3x)^{\frac{3}{2}} - \frac{8}{5}(2 - 3x)^{\frac{5}{2}} + \frac{2}{7}(2 - 3x)^{\frac{7}{2}} \right] + C$

(5) A 13 ft ladder is leaning against a house when its base starts to slide away. By the time the base is 12 ft from the house, the base is moving at the rate of 5 ft/sec.

(a) How fast is the top of the ladder sliding down the wall then?

(b) At what rate is the area of the triangle formed by the ladder, wall, and the ground changing then?

(c) At what rate is the angle $\theta$ between the ladder and the ground changing then?

Answers:

(a) 12 ft/sec

(b) $(-119/2)$ squnits/sec

(c) $-1$ rad/sec
(6) Consider the definite integral \( \int_0^5 (2 - 5x) \, dx \).

(a) Use the definition of the definite integral to evaluate the integral. HINT: 
\[ \sum_{i=1}^{n} i = \frac{n(n+1)}{2}. \]

(b) Use the Fundamental Theorem of Calculus, Part 2 to evaluate the integral

(c) Evaluate the integral by representing the integral in terms of areas.

Answer: \(-105/2\)