Math 114 Midterm Review

Sections covered: Appendix A-D, 1.1, 1.3, 2.1, 2.2, 2.3, 2.4, 2.5, 2.6, 3.1, 3.2, 3.3, 3.5, 3.6, 3.7

(1) Use the definition of derivative to compute the tangent line of the curve $y = 1/x^2$ at the point $(2, 1/4)$.

(2) (a) Let $f(x) = |\sin(x)|$. Find where $f(x)$ is continuous and where $f(x)$ is differentiable. Justify your answer.
    (b) Let $f(x) = \sin(|x|)$. Find where $f(x)$ is continuous and where $f(x)$ is differentiable. Justify your answer.

(3) Let

$$f(x) = \begin{cases} 
-1 - 2x & \text{if } x < -1 \\
x^2 & \text{if } -1 \leq x \leq 1 \\
x & \text{if } x > 1 
\end{cases}$$

Sketch the graph of $f(x)$ and find where $f(x)$ is continuous and where $f(x)$ is differentiable.

(4) Use Intermediate Value Theorem to show that the equation $\tan(x) = 2x$ has at least one solution in the interval $(0, \pi/2)$.

(5) Find the following limits.
    (a) $\lim_{x \to 1} \frac{x - 1}{x^3 - 1}$
    (b) $\lim_{x \to -2} \frac{x - 1}{x^3 - 1}$
    (c) $\lim_{x \to 0} \frac{\sin(3x)}{\tan(2x)}$
    (d) $\lim_{x \to 0} \frac{1}{x} \left( \frac{1}{3} - \frac{1}{3 + x} \right)$
    (e) $\lim_{x \to 0} x \cos \left( 1 + \frac{1}{x} \right)$

(6) Compute

$$\lim_{x \to 1} \frac{\sqrt{x} - 1}{x - 1}$$

by writing it as the derivative of some function $f(x)$ at some number $a$.

(7) A table of values for $f$, $g$, $f'$, $g'$ is given as follows.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$f(x)$</th>
<th>$g(x)$</th>
<th>$f'(x)$</th>
<th>$g'(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>2</td>
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<tr>
<td>2</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>1</td>
</tr>
</tbody>
</table>

1http://www.math.ualberta.ca/~xichen/math11406w/p1.pdf
(a) If \( h(x) = f(g(x)) \), find \( h'(1) \).
(b) If \( H(x) = g(f(x)) \), find \( H'(1) \).

(8) Find the derivatives of the following functions.

(a) \( f(x) = \frac{x^2 + x + 1}{\sqrt{x^2}} \)
(b) \( f(x) = \frac{\sqrt{x} + 1}{\sqrt{x} - 1} \)
(c) \( f(x) = \sin(x^2) \cos(\sqrt{x}) \)
(d) \( f(x) = \sqrt{x + \sqrt{x}} \)

(9) Let \( F(x) = (f(x))^3 \) and \( G(x) = f(x^3) \). If \( f(1) = 1 \) and \( f'(1) = 2 \), find \( F'(1) \) and \( G'(1) \).

(10) Suppose that \( 4 \cos(x) \sin(y) = 1 \). Find \( dy/dx \) by implicit differentiation.