

Math 114 Midterm Review¹

Sections covered: Appendix A-D, 1.1, 1.3, 2.1, 2.2, 2.3, 2.4, 2.5, 2.6, 3.1, 3.2, 3.3, 3.5, 3.6, 3.7

- (1) Use the definition of derivative to compute the tangent line of the curve $y = 1/x^2$ at the point $(2, 1/4)$.
- (2) (a) Let $f(x) = |\sin(x)|$. Find where $f(x)$ is continuous and where $f(x)$ is differentiable. Justify your answer.
(b) Let $f(x) = \sin(|x|)$. Find where $f(x)$ is continuous and where $f(x)$ is differentiable. Justify your answer.
- (3) Let

$$f(x) = \begin{cases} -1 - 2x & \text{if } x < -1 \\ x^2 & \text{if } -1 \leq x \leq 1 \\ x & \text{if } x > 1 \end{cases}.$$

Sketch the graph of $f(x)$ and find where $f(x)$ is continuous and where $f(x)$ is differentiable.

- (4) Use Intermediate Value Theorem to show that the equation $\tan(x) = 2x$ has at least one solution in the interval $(0, \pi/2)$.
- (5) Find the following limits.
 - (a) $\lim_{x \rightarrow 1} \frac{x-1}{x^3-1}$
 - (b) $\lim_{x \rightarrow 2} \frac{x-1}{x^3-1}$
 - (c) $\lim_{x \rightarrow 0} \frac{\sin(3x)}{\tan(2x)}$
 - (d) $\lim_{x \rightarrow 0} \frac{1}{x} \left(\frac{1}{3} - \frac{1}{3+x} \right)$
 - (e) $\lim_{x \rightarrow 0} x \cos \left(1 + \frac{1}{x} \right)$
- (6) Compute

$$\lim_{x \rightarrow 1} \frac{\sqrt[5]{x} - 1}{x - 1}$$

by writing it as the derivative of some function $f(x)$ at some number a .

- (7) A table of values for f, g, f', g' is given as follows.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
1	1	2	1	2
2	2	1	2	1

¹<http://www.math.ualberta.ca/~xichen/math11406w/p1.pdf>

- (a) If $h(x) = f(g(x))$, find $h'(1)$.
 - (b) If $H(x) = g(f(x))$, find $H'(1)$.
- (8) Find the derivatives of the following functions.
- (a) $f(x) = \frac{x^2 + x + 1}{\sqrt[3]{x^2}}$
 - (b) $f(x) = \frac{\sqrt{x} + 1}{\sqrt{x} - 1}$
 - (c) $f(x) = \sin(x^2) \cos(\sqrt{x})$
 - (d) $f(x) = \sqrt{x + \sqrt{x}}$
- (9) Let $F(x) = (f(x))^3$ and $G(x) = f(x^3)$. If $f(1) = 1$ and $f'(1) = 2$, find $F'(1)$ and $G'(1)$.
- (10) Suppose that $4 \cos(x) \sin(y) = 1$. Find dy/dx by implicit differentiation.