Solution for HW 3 (Math 114 Q1)

2.2

6. (a) 1 (b) 1 (c) 1 (d) 1 (e) 1 (f) 2 (g) DNE (h) 2 (i) DNE (j) 2 (k) DNE (l) 0

30. When $1 < x < 2$, $\sin(\pi x) < 0$ and hence $(x+1)/(x \sin \pi x) < 0$. Therefore,

$$\lim_{x \to 1^+} \frac{x+1}{x \sin \pi x} = -\infty$$

2.3

14.

$$\lim_{x \to 4} \frac{x^2 - 4x}{x^2 - 3x - 4} = \lim_{x \to 4} \frac{x(x-4)}{(x+1)(x-4)} = \lim_{x \to 4} \frac{x}{x+1} = \frac{4}{5}$$

30.

$$\lim_{x \to 1} \frac{\sqrt{x} - x^2}{1 - \sqrt{x}} = \lim_{x \to 1} \frac{x^{1/2}(1 - x^{3/2})}{1 - x^{1/2}} = \lim_{x \to 1} \frac{x^{1/2}(1 - x^{1/2})(1 + x^{1/2} + x)}{1 - x^{1/2}}$$
$$= \lim_{x \to 1} x^{1/2}(1 + x^{1/2} + x) = 3$$

38. Since $-1 \le \sin(2\pi/x) \le 1$, $0 \le \sin^2(2\pi/x) \le 1 \Rightarrow 1 \le 1 + \sin^2(2\pi/x) \le 2 \Rightarrow \sqrt{x} \le \sqrt{x}(1 + \sin^2(2\pi/x)) \le 2\sqrt{x}$. And since $\lim_{x \to 0^+} \sqrt{x} = \lim_{x \to 0^+} 2\sqrt{x} = 0$, $\lim_{x \to 0^+} \sqrt{x}(1 + \sin^2(2\pi/x)) = 0$ by squeeze theorem.

42. Since

$$\lim_{x \to 1.5^+} \frac{2x^2 - 3x}{|2x - 3|} = \lim_{x \to 1.5^+} \frac{x(2x-3)}{2x - 3} = 1.5$$

and

$$\lim_{x \to 1.5^-} \frac{2x^2 - 3x}{|2x - 3|} = \lim_{x \to 1.5^-} \frac{x(2x-3)}{-(2x - 3)} = -1.5$$

the limit does not exit.

48. (a) (i) 0 (ii) 0 (iii) 1 (iv) 4 (v) 6 (vi) DNE

58.

$$\lim_{x \to 2} \frac{\sqrt{6-x} - 2}{\sqrt{3-x} - 1} = \lim_{x \to 2} \left(\frac{(\sqrt{6-x} - 2)(\sqrt{6-x} + 2)}{(\sqrt{3-x} - 1)(\sqrt{3-x} + 1)} \right) \cdot \frac{\sqrt{3-x} + 1}{\sqrt{6-x} + 2}$$
$$= \lim_{x \to 2} \frac{\sqrt{3-x} + 1}{\sqrt{6-x} + 2} = \frac{1}{2}$$

2.5

10. Since

$$\lim_{x \to 4} f(x) = \lim_{x \to 4} (x^2 + \sqrt{7 - x}) = \lim_{x \to 4} x^2 + \lim_{x \to 4} \sqrt{7 - x}$$
$$= (\lim_{x \to 4} x)^2 + \sqrt{\lim_{x \to 4} (7 - x)} = (\lim_{x \to 4} x)^2 + \sqrt{\lim_{x \to 4} 7 - \lim_{x \to 4} x}$$
$$= 16 + \sqrt{3} = f(4)$$

$f(x)$ is continuous at $x = 4$.

20. Since

$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} (1 + x^2) = 2 \text{ and } \lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} (4 - x) = 3$$

$\lim_{x \to 1} f(x)$ does not exist. So $f(x)$ is not continuous at $x = 1$.