Math 114 Final Review[1]

(1) Compute the following limits if they exist.

(a) $\lim_{x\to 0} \frac{\sqrt{x+4}-2}{x}$

(b) $\lim_{x\to -\infty} \frac{\sin(2x)}{x}$

(c) $\lim_{x\to \infty} \frac{1-3x^2+2x^3}{1-x^3}$

(d) $\lim_{\theta\to 0} \frac{\cos\theta - 1}{\sin\theta}$

(2) Find the local and absolute maxima and minima of the function $f(x) = \sin x + \cos x$ on $[-\pi, \pi]$.

(3) (a) Show that the equation $x^3 + x + 3 = 0$ has exactly one real root.

(b) Use Newton's method with initial approximation $x_1 = -1$ to find x_3, the third appromixation to the root of the equation $x^3 + x + 3 = 0$.

(4) Sketch the graph of the function

$$f(x) = \frac{x^2+1}{x+1}$$

You must follow the steps A-H as in Sec. 4.5: (A) Domain (B) Intercepts (C) Symmetry (D) Asmptotes (E) Intervals of Increases and Decreases (F) Local maximum and minimum (G) Concavity and points of inflection (H) Sketch the curve.

(5) Compute the following integrals.

(a) $\int_1^4 \frac{x^2+x+1}{\sqrt{x}} dx$

(b) $\int_0^{13} \frac{dx}{\sqrt[3]{(1+2x)^2}}$

(c) $\int_1^2 x\sqrt{x-1}dx$

(6) Find an equation of the line through the point $(3, 5)$ that cuts off the least area from the first quadrant.

[1] http://www.math.ualberta.ca/~xichen/math11406w/fp.pdf

(7) At noon, Ship A is 100 km west of ship B. Ship A is sailing south at 35 km/h and ship B is sailing north at 25 km/h. How fast is the distance between the ships changing at 4:00 pm?

(8) A fence 8 ft tall runs parallel to a tall building at a distance of 4 ft from the building. What is the length of the shortest ladder that will reach from the ground over the fence to the wall of the building?

(9) Find the tangent line to the curve $x^2 + xy + 2y^2 = 4$ at the point $(1, 1)$.

(10) Let $F(x) = \sqrt{f(x)}$ and $G(x) = f(\sqrt{x})$. If $f(1) = 1$, $f'(1) = 2$ and $f''(1) = 3$, find $F''(1)$ and $G''(1)$.

(11) Consider the integral

$$\int_{-2}^{3} (1 - 2x)dx$$

(a) Write the above integral as a limit of Riemann sums.

(b) Compute the limit you obtained in part (a). You may need the formula:

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$

(c) Verify your answer by computing the integral using Fundamental Theorem of Calculus.

(12) Find the area of the largest rectangle that can be inscribed in a right triangle with legs of lengths a and b if two sides of the rectangle lie along the legs.