Math 114 Final Review[1]

Sections covered: Appendix A-D, 1.1, 1.3, 2.1, 2.2, 2.3, 2.4, 2.5, 2.6, 3.1, 3.2, 3.3, 3.5, 3.6, 3.7, 3.8, 3.9, 4.1, 4.2, 4.3, 4.4, 4.5, 4.7, 4.9, 4.10, 5.1, 5.2, 5.3, 5.4, 5.5

(1) Compute the following limits.
   (a) $\lim_{x\to-\infty}(\sqrt{x^2+x}-\sqrt{x^2-1})$
   (b) $\lim_{x\to1}\dfrac{x^2-3x+2}{x^3-1}$
   (c) $\lim_{x\to-\infty}\dfrac{6x^2+5x}{(1-x)(2x-3)}$
   (d) $\lim_{x\to0}\dfrac{\sqrt[3]{1+x}-1}{x}$
   (e) $\lim_{t\to2}\dfrac{t^{-1}-2^{-1}}{t-2}$
   (f) $\lim_{x\to0}\dfrac{\tan(4x)}{\sin(3x)}$

(2) Find the derivative of each of the following functions.
   (a) $f(x)=x\tan(x^2)+\cos(x^3)$
   (b) $f(x)=\dfrac{x^2+1}{x^2-1}$
   (c) $f(t)=\dfrac{6}{\sqrt[3]{t^5}}$
   (d) $f(x)=\sqrt{\cos(\sin(x))}$

(3) Find local and absolute maxima and minima of the function $f(x)=x^3-3x$ on the interval $[-2,2]$.

(4) Two carts start moving from the same point. One travels south at 60 mi/h and the other travels west at 25 mi/h. At what rate is the distance between the cars increasing two hours later?

(5) A lighthouse is located on a small island 3 km away from the nearest point P on a straight shoreline and its light makes four revolutions per minute. How fast is the beam of light moving along the shoreline when it is 1 km from P?

(6) Use Intermediate Value Theorem and Mean Value Theorem to show that the equation $x^5+x^3+x+1=0$ has exactly one solution.

(7) The top and bottom margins of a poster are each 6 cm and the side margins are each 4 cm. If the area of printed material on the poster is fixed at 384 cm^2, find the dimension of the poster with the smallest area.

(8) A box with an open top is to be constructed from a square piece of cardboard, 3 ft wide, by cutting out a square from each of the four corners and bending up the sides. Find the largest volume that such a box can have.

[1] http://www.math.ualberta.ca/~xichen/math11404f/fp1.pdf

(9) A boat leaves a dock at 2 PM and travels due south at a speed of 20 km/h. Another boat has been heading due east at 15 km/h and reaches the same dock at 3 PM. At what time were the two boats closest together?

(10) Sketch the graphs of each of the following functions. You must follow the steps A-H as in Sec. 4.5: (A) Domain (B) Intercepts (C) Symmetry (D) Asmptotes (E) Intervals of Increases and Decreases (F) Local maximum and minimum (G) Concavity and points of inflection (H) Sketch the curve.
   (a) $f(x) = 8x^2 - x^4$
   (b) $f(x) = \dfrac{1+x^2}{1-x}$

(11) Evaluate the following integrals.
   (a) $\int_0^1 (x^2 - x)dx$
   (b) $\int_0^{\pi/2} (\sin(x) - \cos(x))dx$
   (c) $\int \sqrt[2002]{1+3x}dx$
   (d) $\displaystyle\int \frac{x^2}{(x^3+1)^2}dx$

(12) Express the following integral as a limit of Riemann sums. Do not evaluate the limit.
   (a) $\displaystyle\int_2^6 \frac{x}{1+x^5}dx$
   (b) $\int_0^{2\pi} x^2 \sin(x)dx$

(13) The table gives the values of a function obtained from experiment. Use them to estimate $\int_0^6 f(x)dx$ using midpoint rule with $n = 3$.

x	0	1	2	3	4	5	6
$f(x)$	1	3	2	4	6	2	1

(14) Starting with $x_1 = -1$ use Newton's method to find x_3, the third appromixation to the root of the equation $x^3 + x + 3 = 0$.

(15) For what values of x does the graph of $f(x) = x + \sin(x)$ have a horizontal tangent?

(16) Let $F(x) = \sqrt[3]{f(x)}$ and $G(x) = f(\sqrt[3]{x})$. If $f(1) = 2$ and $f'(1) = 3$, find $F'(1)$ and $G'(1)$.

(17) The displacement of a particle is given by $s(t) = A\cos(Bt + C)$ with A, B, C constants. Find the velocity and acceleration of the particle at time t.