

## Math 114 Midterm I Review<sup>1</sup>

Sections covered: Appendix A-D, 1.1, 1.3, 2.1, 2.2, 2.3, 2.4, 2.5, 2.6, 3.1, 3.2, 3.3, 3.5, 3.6

- (1) Use the definition of derivative to compute the tangent line of the curve  $y = 1/x$  at the point  $(2, 1/2)$ .  
(2) Let

$$f(x) = \begin{cases} x^2 & \text{if } x \leq 2 \\ mx + b & \text{if } x > 2 \end{cases}$$

Find the values of  $m$  and  $b$  that make  $f$  differentiable everywhere.

- (3) Let

$$f(x) = \begin{cases} -1 - 2x & \text{if } x < -1 \\ x^2 & \text{if } -1 \leq x \leq 1 \\ x & \text{if } x > 1 \end{cases}$$

Sketch the graph of  $f(x)$  and find where  $f(x)$  is continuous and where  $f(x)$  is differentiable.

- (4) Use Intermediate Value Theorem to show that the equation  $\tan(x) = 2x$  has at least one solution in the interval  $(0, \pi/2)$ .  
(5) Find the following limits.

(a)  $\lim_{x \rightarrow 1} \frac{x-1}{x^3-1}$

(b)  $\lim_{x \rightarrow 2} \frac{x-1}{x^3-1}$

(c)  $\lim_{x \rightarrow 0} \frac{\sin(3x)}{\tan(2x)}$

(d)  $\lim_{x \rightarrow 0} \frac{1}{x} \left( \frac{1}{3} - \frac{1}{3+x} \right)$

(e)  $\lim_{x \rightarrow 0} x \cos \left( 1 + \frac{1}{x} \right)$

- (6) Compute

$$\lim_{x \rightarrow 1} \frac{\sqrt[5]{x} - 1}{x - 1}$$

by writing it as the derivative of some function  $f(x)$  at some number  $a$ .

- (7) (20 points) A table of values for  $f$ ,  $g$ ,  $f'$ ,  $g'$  is given as follows.

---

<sup>1</sup><http://www.math.ualberta.ca/~xichen/math11403f/p1.pdf>

$x$	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
1	1	2	1	2
2	2	1	2	1

- (a) If  $h(x) = f(g(x))$ , find  $h'(1)$ .
- (b) If  $H(x) = g(f(x))$ , find  $H'(1)$ .
- (8) Find the first derivatives of the following functions.
- (a)  $f(x) = \frac{x^2 + x + 1}{\sqrt[3]{x^2}}$
- (b)  $f(x) = \frac{\sqrt{x} + 1}{\sqrt{x} - 1}$
- (c)  $f(x) = \sin(x^2) \cos(\sqrt{x})$
- (d)  $f(x) = \sqrt{x + \sqrt{x}}$
- (9) Let  $F(x) = (f(x))^3$  and  $G(x) = f(x^3)$ . If  $f(1) = 1$  and  $f'(1) = 2$ , find  $F'(1)$  and  $G'(1)$ .