

PRINT NAME: _____

STUDENT ID NUMBER: _____

- (1) No books, notes and calculators are allowed.
- (2) Show your work in details.

Problem	Points	Score
1	15	
2	15	
3	10	
4	20	
5	20	
6	20	
Total	100	

(1) (15 points) Let

$$f(x) = \begin{cases} -2x & \text{if } x \leq -1 \\ x^2 & \text{if } -1 < x < 1 . \\ 2x - 1 & \text{if } x \geq 1 \end{cases}$$

(a) Sketch the graph of $y = f(x)$.

(b) Find where $f(x)$ is continuous and where $f(x)$ is differentiable. Briefly state your reason.

- (2) (15 points) Use Intermediate Value Theorem to show that the equation $x^3 - x + 1 = 0$ has at least one solution.

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(3) (10 points) Compute

$$\lim_{h \rightarrow 0} \frac{\sqrt[4]{16+h} - 2}{h}$$

by writing it as the derivative of some function $f(x)$ at some number a .

(4) (20 points) A table of values for f , g , f' , g' is given as follows.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
1	3	2	4	6
2	1	8	5	7
3	7	2	7	9

(a) If $h(x) = f(g(x))$, find $h'(1)$.

(b) If $H(x) = g(f(x))$, find $H'(1)$.

(5) (20 points) Find the following limits if they exist.

(a) $\lim_{x \rightarrow 2} \frac{x^2 - 4}{x - 2}$

(b) $\lim_{x \rightarrow 1} \frac{x^2 - 4}{x - 2}$

(c) $\lim_{x \rightarrow 0} \frac{\sin(3x)}{\sin(2x)}$

(d) $\lim_{x \rightarrow 0} x \sin\left(\frac{1}{x}\right)$

(6) (20 points) Find the first derivatives of the following functions.

(a) $f(t) = t^2 + 2t - 1$

(b) $f(x) = \frac{x + 1}{x - 1}$

(c) $f(x) = \sin(\cos(\sqrt{x}))$

(d) $f(x) = \sqrt{x\sqrt{x}}$