Math 114 Final Review[1]

Sections covered: Appendix A-D, 1.1, 1.3, 2.1, 2.2, 2.3, 2.4, 2.5, 2.6, 3.1, 3.2, 3.3, 3.5, 3.6, 3.7, 3.8, 3.9, 4.1, 4.2, 4.3, 4.4, 4.5, 4.7, 4.9, 4.10, 5.1, 5.2, 5.3, 5.4, 5.5

(1) Compute the following limits.
   (a) $\lim_{x\to\infty}(\sqrt{x^2+1}-\sqrt{x^2-1})$
   (b) $\lim_{x\to 1}\dfrac{x^3-x}{x^3-1}$
   (c) $\lim_{x\to-\infty}\dfrac{6x^2+5x}{(1-x)(2x-3)}$
   (d) $\lim_{x\to 0}\dfrac{\sqrt{1-x}-\sqrt{1+x}}{x}$
   (e) $\lim_{t\to 2}\dfrac{t^{-1}-2^{-1}}{t-2}$
   (f) $\lim_{x\to 0}\dfrac{\sin(4x)}{\tan(3x)}$

(2) Find the derivative of each of the following functions.
   (a) $f(x) = x\tan(x) + \cos(x^2)$
   (b) $f(x) = \dfrac{x^3+1}{x^3-1}$
   (c) $f(t) = \dfrac{6}{\sqrt[3]{t^5}}$
   (d) $f(x) = \sqrt{\cos(\sin(x))}$

(3) Find local and absolute maxima and minima of the function $f(x) = x^3 - 3x$ on the interval $[-2, 2]$.

(4) A spotlight on the ground shines on a wall 12 m away. If a man 2 m tall walks from the spotlight toward the building at a speed of 1.6 m/s, how fast is his shadow on the building decreasing when he is 4 m from the building?

(5) Use Intermediate Value Theorem and Mean Value Theorem to show that the equation $x^5 + x^3 + x + 1 = 0$ has exactly one solution.

(6) The top and bottom margins of a poster are each 6 cm and the side margins are each 4 cm. If the area of printed material on the poster is fixed at 384 cm^2, find the dimension of the poster with the smallest area.

(7) A cylindrical can is made out of materials costing 6 dollar/m^2 for the top and bottom and 10 dollar/m^2 for the sides. Assume

[1] http://www.math.ualberta.ca/~xichen/math11403f/fp1.pdf

that its volume is fixed at 10 m^3. Find the dimension that minimizes the cost.

(8) Two cars start moving from the same point. One travels south at 60 mph and the other travels west at 25 mph. At what rate is the distance between the cars increasing two hours later?

(9) Sketch the graphs of each of the following functions. You must follow the steps A-H as in Sec. 4.5: (A) Domain (B) Intercepts (C) Symmetry (D) Asmptotes (E) Intervals of Increases and Decreases (F) Local maximum and minimum (G) Concavity and points of inflection (H) Sketch the curve.

(a) $f(x) = 8x^2 - x^4$

(b) $f(x) = \dfrac{1+x}{1-x}$

(10) Evaluate the following integrals.

(a) $\int_0^1 (x^2 - x)dx$

(b) $\int_0^{\pi/2} (\sin(x) - \cos(x))dx$

(c) $\int \sqrt[2002]{1+3x}dx$

(d) $\displaystyle\int \frac{x^2}{(x^3+1)^2}dx$

(11) Express the following integral as a limit of Riemann sums. Do not evaluate the limit.

(a) $\displaystyle\int_2^6 \frac{x}{1+x^5}dx$

(b) $\int_0^{2\pi} x^2 \sin(x)dx$

(12) The table gives the values of a function obtained from experiment. Use them to estimate $\int_0^6 f(x)dx$ using midpoint rule with $n = 3$.

x	0	1	2	3	4	5	6
$f(x)$	1	3	2	4	6	2	1

(13) Starting with $x_1 = -1$ use Newton's method to find x_3, the third appromixation to the root of the equation $x^3 + x + 3 = 0$.

(14) For what values of x does the graph of $f(x) = x + \sin(x)$ have a horizontal tangent?

(15) Let $F(x) = \sqrt[3]{f(x)}$ and $G(x) = f(\sqrt[3]{x})$. If $f(1) = 2$ and $f'(1) = 3$, find $F'(1)$ and $G'(1)$.

(16) The displacement of a particle is given by $s(t) = A\cos(Bt + C)$ with A, B, C constants. Find the velocity and acceleration of the particle at time t.