

## Sample Midterm<sup>1</sup>

- (1) (10 points) Let

$$f(x) = \begin{cases} cx - 1 & \text{if } x \geq 1 \\ 1 - cx^2 & \text{if } x < 1 \end{cases},$$

where  $c$  is a constant. For what values of  $c$  is  $f(x)$  continuous on  $(-\infty, \infty)$ ?

- (2) (15 points) Water is leaking out of an inverted conical tank at a rate of  $10,000 \text{ cm}^3/\text{min}$  at the same time that water is being pumped into the tank at a constant rate. The tank has height 6 m and the diameter at the top is 4 m. If the water level is rising at a rate of  $20 \text{ cm}/\text{min}$  when the height of the water is 2 m, find the rate at which water is being pumped into the tank.
- (3) (10 points) Find the tangent line of the curve  $y^2 = x^3$  at the point  $(1, 1)$ . (Do not use the laws of derivative to find the slope. Compute it using its definition.)
- (4) (20 points) Find the following limits.
- (a)  $\lim_{x \rightarrow 1} \frac{x - 1}{x^3 - 1}$
  - (b)  $\lim_{x \rightarrow 2} \frac{x - 1}{x^3 - 1}$
  - (c)  $\lim_{x \rightarrow 0} \frac{\sin(3x)}{\tan(2x)}$
  - (d)  $\lim_{x \rightarrow 0} \frac{1}{x} \left( \frac{1}{3} - \frac{1}{3+x} \right)$
  - (e)  $\lim_{x \rightarrow 1^-} \frac{|x^2 - 1|}{x^2 - 1}$
- (5) (20 points) Find the first derivatives of the following functions.
- (a)  $f(x) = \frac{x^2 + x + 1}{\sqrt[3]{x^2}}$
  - (b)  $f(x) = \frac{\sqrt{x} + 1}{\sqrt{x} - 1}$
  - (c)  $f(x) = \sin(x^2) \cos(\sqrt{x})$
  - (d)  $f(x) = \sqrt{x + \sqrt{x}}$
- (6) (10 points) Let  $F(x) = (f(x))^3$  and  $G(x) = f(x^3)$ . If  $f(1) = 1$  and  $f'(1) = 2$ , find  $F'(1)$  and  $G'(1)$ .
- (7) (15 points) Find the tangent line of the curve  $\sin x + \sin y = 1$  at the point  $(0, \pi/2)$ .

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<sup>1</sup><http://www.math.ualberta.ca/~xichen/math11402f/p1.pdf>