

PRINT NAME: _____

STUDENT ID NUMBER: _____

- (1) No books, notes and calculators are allowed.
- (2) Show your work in details.

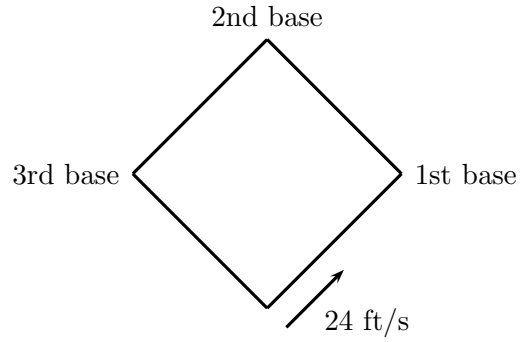
Problem	Points	Score
1	10	
2	15	
3	10	
4	20	
5	20	
6	10	
7	15	
Total	100	

(1) (10 points) Let c be a constant and

$$f(x) = \begin{cases} x - c & \text{if } x \geq 1 \\ c - 3x & \text{if } x < 1 \end{cases}.$$

For what values of c is $f(x)$ continuous everywhere on $(-\infty, \infty)$? You must justify your answer. Also sketch the graph of $f(x)$ when it is continuous.

- (2) (15 points) A baseball diamond is a square with side 90 ft. A batter hits the ball and runs toward first base with a speed of 24 ft/s.



- (a) At what rate is his distance from the second base decreasing when he is halfway to first base?
- (b) At what rate is his distance from third base increasing at the same moment?

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- (3) (10 points) Find the tangent line of the curve $y = \sqrt{x}$ at the point $(4, 2)$. (Do not use the laws of derivative to find the slope. Compute it using its definition.)

(4) (20 points) Find the following limits if they exist.

(a) $\lim_{x \rightarrow -3} \frac{x^2 - x - 12}{x + 3}$

(b) $\lim_{x \rightarrow \pi/4} \frac{\sin 2x}{\tan x}$

(c) $\lim_{x \rightarrow 0} \frac{\sin(3x)}{\sin(2x)}$

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$$(d) \lim_{x \rightarrow 1} \left(\frac{1}{x-1} - \frac{2}{x^2-1} \right)$$

$$(e) \lim_{x \rightarrow 4^-} \frac{|x+4|}{x+4}$$

(5) (20 points) Find the first derivatives of the following functions.

(a) $f(t) = \sqrt[3]{t^2} + 2\sqrt{t^3}$

(b) $f(x) = \frac{x^5}{x^3 - 1}$

(c) $f(x) = \sin(\cos(\sqrt{x}))$

(d) $f(x) = \sqrt{x\sqrt{x}}$

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- (6) (10 points) Let $F(x) = \sqrt{f(x)}$ and $G(x) = f(\sqrt{x})$. If $f(1) = 2$ and $f'(1) = 3$, find $F'(1)$ and $G'(1)$.

(7) (15 points) Find the tangent line of the curve

$$2 \sin x \cos y = 1$$

at the point $(\pi/4, \pi/4)$.