Sample Final

(1) (30 points) Compute the following limits.
   (a) $\lim_{x\to 1} \dfrac{x^2-2x+1}{x^3-1}$
   (b) $\lim_{x\to \infty} \dfrac{x^2-2x+1}{x^3-1}$
   (c) $\lim_{x\to \infty} (\sqrt{x^2+x}-\sqrt{x^2-1})$
   (d) $\lim_{x\to 0} \dfrac{\sqrt{3+x}-\sqrt{3}}{x}$
   (e) $\lim_{t\to 0} \left(\dfrac{1}{t\sqrt{1+t}}-\dfrac{1}{t}\right)$
   (f) $\lim_{x\to 0} \dfrac{\sin(2x)}{\sin(3x)}$

(2) (20 points) Find the derivative of each of the following functions.
   (a) $f(x)=\cos(x\tan x)$
   (b) $f(x)=\dfrac{x^2}{x^3+x+1}$
   (c) $f(x)=(\sin x)\sqrt{x}$
   (d) $f(x)=\sqrt{x+\sqrt[3]{x}}$

(3) (20 points) Find local and absolute maxima and minima of the function $f(x)=x^3-3x+1$ on the interval $[-3,2]$.

(4) (20 points) A street light is mounted at the top of a 15-ft-tall pole. A man 6 ft tall walks away from the pole with a speed of 5 ft/s along a straight path. How fast is the tip of his shadow moving when he is 40 ft from the pole?

(5) (20 points) If 600 $\text{cm}^2$ of material is available to make a box with a square base and and open top. Find the largest possible volume of the box.

(6) (20 points) A poster is to have an area of 180 $\text{in}^2$ with 1-inch margins at the bottom and siders and a 2-inch margin at the top. What dimensions will give the largest printed area?

(7) (20 points) Water is leaking out of an inverted conical tank at a rate of $10,000$ $\text{cm}^3/\text{min}$ at the same time that water is being pumped into the tank at a constant rate. The tank has height 6 m and the diameter at the top is 4 m. If the water level is

rising at a rate of 20 cm/min when the height of the water is 2 m, find the rate at which water is being pumped into the tank.

(8) (30 points) Sketch the graphs of each of the following functions.
   (a) $f(x) = x^3 - 9x$
   (b) $f(x) = \sqrt{x+1} - \sqrt{x}$

(9) (20 points) Evaluate the following integrals.
   (a) $\int (x^3 + x)dx$
   (b) $\int_0^{\pi/2} (\sin x + \cos x)dx$
   (c) $\int_0^1 \sqrt[3]{1+3x}dx$
   (d) $\int x \sin(1+x^2)dx$