MATHEMATICS 113/114
Final Examination
Fall 2014

Date: Tuesday, December 9, 2014
Time: 14:00-16:00

LAST NAME: ___________________ FIRST NAME: ___________________
(Please, print!)

Please, check your section/instructor!

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<td>Xi Chen</td>
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Instructions

1. This is a closed book exam. No books, notes, calculators or other electronic devices are allowed!

2. Show all your work.

3. Write your answers in the spaces provided. If you need more room, write on the back of the page, clearly indicating that you are doing so.

4. Partial credits will not be given for multiple choice questions.
5. Use last page for work you do NOT want to be graded.
6. Make sure your examination paper has ten multiple choice questions and five long answer problems.

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Multiple Choice Question

Please enter your answer as a CAPITAL LETTER in the BOX provided.

1. The slope of the tangent line to the curve defined by the equation \( x^2y^3 - x^3y^2 = 12 \) at the point \((-1, 2)\) is equal to:
   (A) 0; (B) \(-\frac{4}{5}\); (C) 1; (D) \(\frac{7}{4}\);

2. Two points on the graph of \( f(x) = x + \frac{1}{x} \) where the tangent line is parallel to the secant line through \( A(1/2, 5/2) \) and \( B(2, 5/2) \) have \( x \)-coordinates equal to:
   (A) \(-1, 1\);
   (B) \(\sqrt{2}, -\sqrt{2}\);
   (C) \(3, -3\);
   (D) \(\sqrt{3}, -\sqrt{3}\);

3. If \( f'(4) = 3 \), then \( \lim_{x \to 4} \frac{f(x) - f(4)}{\sqrt{x} - 2} = \)
   (A) 22 (B) 1 (C) 12 (D) 0
4. If \( f'(0) = 5 \), \( u(x) = f(x^2 + ax) \) and \( u'(0) = 5 \), then the constant “a” is:
   (A) 0; (B) 1; (C) 2; (D) 3;

5. The absolute minimum and the absolute maximum values of \( f(t) = 3t^5 - 20t^3 \) on \([-1, 2]\) are respectively:
   (A) \(-1, 1\); (B) \(-64, 17\); (C) \(-64, 64\); (D) \(0, 17\);

6. \( \lim_{{x \to 0}} \frac{\tan(6x)}{2x^2 + 3x} = \)
   (A) 1; (B) 2; (C) 3; (D) \(\sqrt{3}/2\);
7. If \( g(x) = e^{3x^2 + 1} \int_1^{x^4} (t^2 + 1)^2 \, dt \), then \( g'(1) \) equals:
(A) 11e;  (B) 2e;  (C) 12e^4;  (D) e^2;

8. The total area between the graph of the function \( f(x) = |2 - x| \) and the \( x \)-axis on \([0, 5]\) is
(A) 15/2;  (B) 8;  (C) 13/2;  (D) 50/3;

9. If \( f''(x) = 2 + \cos x \), \( f(0) = -1 \) and \( f(\pi/2) = 0 \) then \( f(3\pi/2) \) equals:
(A) \( \pi^2/4 \);  (B) \(-4\pi^2/9\);  (C) 1;  (D) \( 3\pi^2/2 \);

10. \( \int_0^2 \sqrt{4 - x^2} \, dx = \)
(A) \( \pi \);  (B) 2\( \pi \);  (C) 3\( \pi/2 \);  (D) 3\( \pi \);
Long Answer Problems. Show all your work.

1. (15 points) Evaluate the following integrals:

   (a) (5 points) \[ \int_{0}^{3} |x^2 - 3x + 2| \, dx \]

   (b) (5 points) \[ \int_{0}^{1} \frac{e^x}{e^x + 2} \, dx \]

   (c) (5 points) \[ \int \frac{\cos(\sqrt{x})}{\sqrt{x}} \, dx \]
2. (20 points) Let \( f(x) = \frac{2 + x - x^2}{(x - 1)^2} \). Given that \( f'(x) = \frac{x - 5}{(x - 1)^3} \) and \( f''(x) = \frac{14 - 2x}{(x - 1)^4} \), find each of the following:

(a) (2 points) The domain of \( f \) and intercepts with \( x \) and \( y \) axes.
(b) (4 points) Vertical and horizontal asymptotes.
(c) (4 points) The intervals of increase and decrease.
(d) (4 points) The local maximum and minimum values.
(e) (4 points) The intervals of concavity and inflection points.
(f) (2 points) Sketch the graph of \( f \).

Show all your work and justify each answer.
3. (10 points) A kite 100 ft above the ground moves horizontally at a speed of 8 ft/s. At what rate is the angle between the string and the ground changing when 200 ft of string has been let out?
4. (10 points) You need to fence in a rectangular play zone for children to fit into a right-
triangular plot with sides measuring 4 m and 12 m. Two sides of the rectangle should 
be on the sides and one vertex on the hypotenuse of the triangle. What is the maximum 
area for this play zone?
5. (15 points) Do the following:

(a) (7 points) Evaluate the integral \( \int_{0}^{3} \left( \frac{x^2}{9} + 1 \right) dx \) using the definition of definite integrals. Use right end points. No marks will be given if the definition is not used.

\[ \text{Hint: } \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \]

(b) (2 points) Evaluate the integral \( \int_{0}^{3} \left( \frac{x^2}{9} + 1 \right) dx \) using the Fundamental Theorem of Calculus.

(c) (6 points) Show that the equation \( x^5 + 2x + 1 = 0 \) has exactly one solution.