Galois connections between generating systems of sets and sequences

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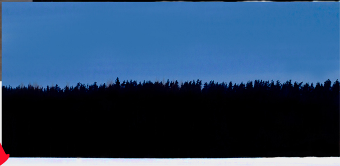
POSITIVE

GREETINGS

FROM



ESTONIA

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- Show that there is a deep relationship between these notions

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- Given two generating systems of sets, one obtains an operator ideal by considering all of the operators that map the sets of the first system to the sets of the second system
- Generating systems of sequences produce generating systems of sets

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Why?

- Simplifies notation
- Avoids leaving ZFC
- All results still hold in the general context

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Example: $\mathbf{K} \in \mathrm{GSet}$, $\mathbf{B} \in \mathrm{GSet}$

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Examples: $\mathbf{c} \in \mathrm{GSeq}$, $\mathbf{m} \in \mathrm{GSeq}$

Mapping systems of sequences to systems of sets

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By definition, $\Psi(\mathbf{c}) = \mathsf{K}$.
Also, $\Psi(\mathbf{m}) = \mathsf{B}$.

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Definition

A sequence (x_k) belongs to the system of sequences $\Phi(\mathbf{G})$ iff it is contained in G for some $G \in \mathbf{G}$.

An alternative notion of relative compactness

A. Grothendieck proved the following result in his famous Memoir.

Theorem (Grothendieck, 1955)

A set $K \subset X$ is relatively compact if and only if there exists $x = (x_k) \in c_0(X)$ so that $K \subset \{\sum_{k=1}^{\infty} \alpha_k x_k \mid (\alpha_k) \in B_{\ell_1}\}$.

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Let us use the shorthands $\ell_\infty(X) := c_0(X)$ and $\ell_\infty := c_0$.

Definition (Ain, Lillemets, Oja, 2012)

Let $1 \leq p \leq \infty$ and $1 \leq r \leq p^*$.
A set $K \subset X$ is said to be *relatively* (p, r)*-compact*, if there exists $x = (x_k) \in \ell_p(X)$ so that $K \subset \{\sum_{k=1}^{\infty} \alpha_k x_k \mid (\alpha_k) \in B_{\ell_r}\}$.

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Denote by $\mathbf{K}_{(p,r)}$ the set of all relatively (p, r)-compact sets in X.
Observe that $\mathbf{K}_{(\infty,1)} = \mathbf{K}$.

A question

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Questions

Which generating systems of sets are sequentially generatable? For example, is $\mathbf{K}_{(p,r)}$ sequentially generatable?

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Let A and B be ordered sets.

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Lemma

Assume that the pair (R, S) is a Galois connection between ordered sets A and B. Let $a, a_1, a_2 \in A$ and $b, b_1, b_2 \in B$. Then

(1) $a \leq SR(a)$ and $RS(b) \leq b$;

(2) $a_1 \leq a_2 \Rightarrow R(a_1) \leq R(a_2)$ and $b_1 \leq b_2 \Rightarrow S(b_1) \leq S(b_2)$;

(3) $R(a) = RSR(a)$ and $S(b) = SRS(b)$.

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(3) $R(a) = RSR(a)$ and $S(b) = SRS(b)$.

For a given $a \in A$ there exists $b \in B$ such that $S(b) = a$ if and only if $a = SR(a)$.

Defining orders on systems of sets and sequences

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Relation $\sim$ is an equivalence relation on GSeq. Preorder $\lesssim$ on GSeq induces an order on $\mathrm{GSeq}/_\sim$ via $[\mathbf{g}] \leq [\mathbf{h}]$ whenever $\mathbf{g} \lesssim \mathbf{h}$.

Maps between GSet and $\mathrm{GSeq}/_{\sim}$

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$$\phi(\mathbf{G}) := \left[\Phi(\mathbf{G})\right], \text{ where } \mathbf{G} \in \mathrm{GSet}.$$

Define the operator $\psi \colon \mathrm{GSeq}/_{\sim} \to \mathrm{GSet}$ by

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For example, $\psi([\mathbf{c}]) = \Psi(\mathbf{c}) = \mathbf{K}$.

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Let $1 \leq p \leq \infty$ and let $1 \leq r \leq p^*$. Then $\Psi\left(\Phi\left(\mathbf{K}_{(p,r)}\right)\right) = \mathbf{K}$.

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Theorem (Delgado, Piñeiro, Serrano, 2010)

Let $1 \leq p < \infty$. If X is infinite dimensional, then there exists a compact set $K \subseteq X$ such that $K \notin \mathbf{K}_{(p,r)}$.

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To conclude:

- The notion of a generating system of sets is a useful tool for generating new operator ideals.
- There is an intimate relationship between generating systems of sets and sequences.
- Not all generating systems of sets can be created from systems of sequences – but at least we have a criterion for deciding which of them can be.

This talk was based on a paper "Galois connections between generating systems of sets and sequences", which was published (online) in 2016 in the journal Positivity.

Thank you for listening!