Orthogonally additive polynomials on Riesz spaces

ELMILOUD CHIL

Positivity XI
University of Alberta, Edmonton, Canada
July 17-21, 2017

- A multilinear mapping $T : E^n \longrightarrow F$ is said to be orthosymmetric if $T(x_1, ..., x_n) = 0$ whenever $x_1, ..., x_n \in E$ satisfy $x_i \perp x_j$ for some $i \neq j$.
- Let E be a vector lattice and let F be a topological space. A map $P : E \to F$ is called a homogeneous polynomial of degree n (or a n-homogeneous polynomial) if $P(x) = \psi(x, .., x)$, where ψ is a n-multilinear map from E^n into F.
- A homogeneous polynomial, of degree n, $P : E \to F$ is said to be orthogonally additive if $P(x + y) = P(x) + P(y)$ where $x, y \in E$ are orthogonally (i.e. $|x| \wedge |y| = 0$).
- We denote by $\mathcal{P}_0({}^nE, F)$ the set of n-homogeneous orthogonally additive polynomials from E to F.

- A multilinear mapping $T : E^n \longrightarrow F$ is said to be orthosymmetric if $T(x_1, ..., x_n) = 0$ whenever $x_1, ..., x_n \in E$ satisfy $x_i \perp x_j$ for some $i \neq j$.
- Let E be a vector lattice and let F be a topological space. A map $P : E \to F$ is called a homogeneous polynomial of degree n (or a n-homogeneous polynomial) if $P(x) = \psi(x, .., x)$, where ψ is a n-multilinear map from E^n into F.
- A homogeneous polynomial, of degree n, $P : E \to F$ is said to be orthogonally additive if $P(x + y) = P(x) + P(y)$ where $x, y \in E$ are orthogonally (i.e. $|x| \wedge |y| = 0$).
- We denote by $\mathcal{P}_0(^nE, F)$ the set of n-homogeneous orthogonally additive polynomials from E to F.

- A multilinear mapping $T : E^n \longrightarrow F$ is said to be orthosymmetric if $T(x_1, ..., x_n) = 0$ whenever $x_1, ..., x_n \in E$ satisfy $x_i \perp x_j$ for some $i \neq j$.
- Let E be a vector lattice and let F be a topological space. A map $P : E \rightarrow F$ is called a homogeneous polynomial of degree n (or a n-homogeneous polynomial) if $P(x) = \psi(x, .., x)$, where ψ is a n-multilinear map from E^n into F.
- A homogeneous polynomial, of degree n, $P : E \rightarrow F$ is said to be orthogonally additive if $P(x + y) = P(x) + P(y)$ where $x, y \in E$ are orthogonally (i.e. $|x| \wedge |y| = 0$).
- We denote by $\mathcal{P}_0({}^nE, F)$ the set of n-homogeneous orthogonally additive polynomials from E to F.

- A multilinear mapping $T : E^n \longrightarrow F$ is said to be orthosymmetric if $T(x_1, ..., x_n) = 0$ whenever $x_1, ..., x_n \in E$ satisfy $x_i \perp x_j$ for some $i \neq j$.
- Let E be a vector lattice and let F be a topological space. A map $P : E \to F$ is called a homogeneous polynomial of degree n (or a n-homogeneous polynomial) if $P(x) = \psi(x, .., x)$, where ψ is a n-multilinear map from E^n into F.
- A homogeneous polynomial, of degree n, $P : E \to F$ is said to be orthogonally additive if $P(x + y) = P(x) + P(y)$ where $x, y \in E$ are orthogonally (i.e. $|x| \wedge |y| = 0$).
- We denote by $\mathcal{P}_0(^nE, F)$ the set of n-homogeneous orthogonally additive polynomials from E to F.

Introduction

One of the relevent problems in Operator Theory is to describe orthogonally additive polynomials via linear operators. This problem can be treated in different manner, depending on domains and codomains on which polynomials act. Interest in orthogonally additive polynomials on Banach lattices originates in the work of **Sundaresan**, where the space of n-homogeneous orthogonally additive polynomials on the Banach lattices l_p and $L_p\,[0,1]$ was characterized. It is only recently that the class of such mappings have been getting more attention. We are thinking here about works on orthogonally additive polynomials and holomorphic functions and orthosymmetric multilinear mappings on different Banach lattices and also $\mathbb{C}^*$-algebras. Proofs of the aforementioned results are strongly based on the representation of this spaces as vector spaces of extended continuous functions. So they are not applicable to general Riesz spaces. That is why we need to develop new approaches. Actually, the innovation of this work consist in making a relationship between orthogonally additive homogeneous polynomilas and orthosymmetric multilinear mappings which leads to a constructively proofs of **Sundaresan** results.

Introduction

One of the relevent problems in Operator Theory is to describe orthogonally additive polynomials via linear operators. This problem can be treated in different manner, depending on domains and codomains on which polynomials act. Interest in orthogonally additive polynomials on Banach lattices originates in the work of **Sundaresan**, where the space of n-homogeneous orthogonally additive polynomials on the Banach lattices l_p and $L_p\,[0,1]$ was characterized. It is only recently that the class of such mappings have been getting more attention. We are thinking here about works on orthogonally additive polynomials and holomorphic functions and orthosymmetric multilinear mappings on different Banach lattices and also $\mathbb{C}^*$-algebras. Proofs of the aforementioned results are strongly based on the representation of this spaces as vector spaces of extended continuous functions. So they are not applicable to general Riesz spaces. That is why we need to develop new approaches. Actually, the innovation of this work consist in making a relationship between orthogonally additive homogeneous polynomilas and orthosymmetric multilinear mappings which leads to a constructively proofs of **Sundaresan** results.

Historical

- **1991** : **Sundaresan** On ℓ_p and $L_p[0,1]$

$$P(f) = \int f^n g d\mu.$$

- **2005 : Garcia , Villaneva, Carando, Lassale, Zalduendo** : On $C(X)$

$$P(f) = \int_X f^n g d\mu.$$

- **2006 : Benyamini, Lassale, LIanova** : On Banach lattices
- **2008 : Plazuelos, Peralta, Villanueva** : On C^*-algebras
- **2009 : Ibort, Linares, LIavona :** On ℓ_p
- **2010-2012 : Carando, Lassale, Zalduendo, Jaramillo, Prieto, Zalduendo :** On holomorphic functions on open subsets of $C(X)$
- **2012 Bu, Buskes :** Banach lattices (Tensor products).
- **2014 Chil, Meyer** : On Uniformluy complete Riesz spaces
- **2015 Chil, Mokaddem, Bourokba :** On Riesz spaces

Historical

- **1991** : **Sundaresan** On ℓ_p and $L_p[0,1]$

$$P(f) = \int f^n g d\mu.$$

- **2005 : Garcia , Villaneva, Carando, Lassale, Zalduendo** : On $C(X)$

$$P(f) = \int_X f^n g d\mu.$$

- **2006 : Benyamini, Lassale, LIanova** : On Banach lattices
- **2008 : Plazuelos, Peralta, Villanueva** : On C^*-algebras
- **2009 : Ibort, Linares, LIavona :** On ℓ_p
- **2010-2012 : Carando, Lassale, Zalduendo, Jaramillo, Prieto, Zalduendo :** On holomorphic functions on open subsets of $C(X)$
- **2012 Bu, Buskes :** Banach lattices (Tensor products).
- **2014 Chil, Meyer** : On Uniformluy complete Riesz spaces
- **2015 Chil, Mokaddem, Bourokba :** On Riesz spaces

Historical

- **1991** : **Sundaresan** On ℓ_p and $L_p[0,1]$

$$P(f) = \int f^n g d\mu.$$

- **2005 : Garcia , Villaneva, Carando, Lassale, Zalduendo** : On $C(X)$

$$P(f) = \int_X f^n g d\mu.$$

- **2006 : Benyamini, Lassale, LIanova** : On Banach lattices
- **2008 : Plazuelos, Peralta, Villanueva** : On C^*-algebras
- **2009 : Ibort, Linares, LIavona :** On ℓ_p
- **2010-2012 : Carando, Lassale, Zalduendo, Jaramillo, Prieto, Zalduendo :** On holomorphic functions on open subsets of $C(X)$
- **2012 Bu, Buskes :** Banach lattices (Tensor products).
- **2014 Chil, Meyer** : On Uniformluy complete Riesz spaces
- **2015 Chil, Mokaddem, Bourokba :** On Riesz spaces

Historical

- **1991** : **Sundaresan** On ℓ_p and $L_p[0,1]$

$$P(f) = \int f^n g d\mu.$$

- **2005 : Garcia , Villaneva, Carando, Lassale, Zalduendo** : On $C(X)$

$$P(f) = \int_X f^n g d\mu.$$

- **2006 : Benyamini, Lassale, LIanova** : On Banach lattices
- **2008 : Plazuelos, Peralta, Villanueva** : On C^*-algebras
- **2009 : Ibort, Linares, LIavona : On** ℓ_p
- **2010-2012 : Carando, Lassale, Zalduendo, Jaramillo, Prieto, Zalduendo :** On holomorphic functions on open subsets of $C(X)$
- **2012 Bu, Buskes :** Banach lattices (Tensor products).
- **2014 Chil, Meyer** : On Uniformluy complete Riesz spaces
- **2015 Chil, Mokaddem, Bourokba :** On Riesz spaces

Historical

- **1991** : **Sundaresan** On ℓ_p and $L_p[0,1]$

$$P(f) = \int f^n g d\mu.$$

- **2005 : Garcia , Villaneva, Carando, Lassale, Zalduendo** : On $C(X)$

$$P(f) = \int_X f^n g d\mu.$$

- **2006 : Benyamini, Lassale, LIanova** : On Banach lattices
- **2008 : Plazuelos, Peralta, Villanueva** : On C^*-algebras
- **2009 : Ibort, Linares, LIavona :** On ℓ_p
- **2010-2012 : Carando, Lassale, Zalduendo, Jaramillo, Prieto, Zalduendo :** On holomorphic functions on open subsets of $C(X)$
- **2012 Bu, Buskes :** Banach lattices (Tensor products).
- **2014 Chil, Meyer** : On Uniformluy complete Riesz spaces
- **2015 Chil, Mokaddem, Bourokba :** On Riesz spaces

Historical

- **1991** : **Sundaresan** On ℓ_p and $L_p[0,1]$

$$P(f) = \int f^n g d\mu.$$

- **2005 : Garcia , Villaneva, Carando, Lassale, Zalduendo** : On $C(X)$

$$P(f) = \int_X f^n g d\mu.$$

- **2006 : Benyamini, Lassale, LIanova** : On Banach lattices
- **2008 : Plazuelos, Peralta, Villanueva** : On C^*-algebras
- **2009 : Ibort, Linares, LIavona :** On ℓ_p
- **2010-2012 : Carando, Lassale, Zalduendo, Jaramillo, Prieto, Zalduendo :** On holomorphic functions on open subsets of $C(X)$
- **2012 Bu, Buskes :** Banach lattices (Tensor products).
- **2014 Chil, Meyer** : On Uniformluy complete Riesz spaces
- **2015 Chil, Mokaddem, Bourokba :** On Riesz spaces

Historical

- **1991** : **Sundaresan** On ℓ_p and $L_p[0,1]$

$$P(f) = \int f^n g d\mu.$$

- **2005 : Garcia , Villaneva, Carando, Lassale, Zalduendo** : On $C(X)$

$$P(f) = \int_X f^n g d\mu.$$

- **2006 : Benyamini, Lassale, LIanova** : On Banach lattices
- **2008 : Plazuelos, Peralta, Villanueva** : On C^*-algebras
- **2009 : Ibort, Linares, LIavona :** On ℓ_p
- **2010-2012 : Carando, Lassale, Zalduendo, Jaramillo, Prieto, Zalduendo :** On holomorphic functions on open subsets of $C(X)$
- **2012 Bu, Buskes :** Banach lattices (Tensor products).
- **2014 Chil, Meyer** : On Uniformluy complete Riesz spaces
- **2015 Chil, Mokaddem, Bourokba :** On Riesz spaces

Historical

- **1991** : **Sundaresan** On ℓ_p and $L_p[0,1]$

$$P(f) = \int f^n g d\mu.$$

- **2005 : Garcia , Villaneva, Carando, Lassale, Zalduendo** : On $C(X)$

$$P(f) = \int_X f^n g d\mu.$$

- **2006 : Benyamini, Lassale, LIanova** : On Banach lattices
- **2008 : Plazuelos, Peralta, Villanueva** : On C^*-algebras
- **2009 : Ibort, Linares, LIavona :** On ℓ_p
- **2010-2012 : Carando, Lassale, Zalduendo, Jaramillo, Prieto, Zalduendo :** On holomorphic functions on open subsets of $C(X)$
- **2012 Bu, Buskes :** Banach lattices (Tensor products).
- **2014 Chil, Meyer** : On Uniformluy complete Riesz spaces
- **2015 Chil, Mokaddem, Bourokba :** On Riesz spaces

Historical

- **1991** : **Sundaresan** On ℓ_p and $L_p[0,1]$

$$P(f) = \int f^n g d\mu.$$

- **2005 : Garcia , Villaneva, Carando, Lassale, Zalduendo** : On $C(X)$

$$P(f) = \int_X f^n g d\mu.$$

- **2006 : Benyamini, Lassale, LIanova** : On Banach lattices
- **2008 : Plazuelos, Peralta, Villanueva** : On C^*-algebras
- **2009 : Ibort, Linares, LIavona :** On ℓ_p
- **2010-2012 : Carando, Lassale, Zalduendo, Jaramillo, Prieto, Zalduendo :** On holomorphic functions on open subsets of $C(X)$
- **2012 Bu, Buskes :** Banach lattices (Tensor products).
- **2014 Chil, Meyer** : On Uniformluy complete Riesz spaces
- **2015 Chil, Mokaddem, Bourokba :** On Riesz spaces

orthosymmetric multilinear mapping

- A multilinear mapping $T : E^n \longrightarrow F$ is said to be orthosymmetric if $T(x_1, ..., x_n) = 0$ whenever $x_1, ..., x_n \in E$ satisfy $x_i \perp x_j$ for some $i \neq j$..
- A bilinear map $T : E \times E \to F$ is positive if $T(x, y) \geq 0$ whenever $(x, y) \in E^+ \times E^+$, and is order bounded if given $(x, y) \in E^+ \times E^+$ there exists $a \in F^+$ such that $|T(z, w)| \leq a$ for all $(0, 0) \leq (z, w) \leq (x, y) \in E \times E$
- $T : E \times E \to F$ is (r-u) continuous if $x_n, y_n \longrightarrow 0$ (r-u) in E implies that $T(x_n, y_n) \longrightarrow 0$ (r-u) in F.

orthosymmetric multilinear mapping

- A multilinear mapping $T : E^n \longrightarrow F$ is said to be orthosymmetric if $T(x_1, ..., x_n) = 0$ whenever $x_1, ..., x_n \in E$ satisfy $x_i \perp x_j$ for some $i \neq j$..
- A bilinear map $T : E \times E \to F$ is positive if $T(x, y) \geq 0$ whenever $(x, y) \in E^+ \times E^+$, and is order bounded if given $(x, y) \in E^+ \times E^+$ there exists $a \in F^+$ such that $|T(z, w)| \leq a$ for all $(0, 0) \leq (z, w) \leq (x, y) \in E \times E$
- $T : E \times E \to F$ is (r-u) continuous if $x_n, y_n \longrightarrow 0$ (r-u) in E implies that $T(x_n, y_n) \longrightarrow 0$ (r-u) in F.

orthosymmetric multilinear mapping

- A multilinear mapping $T : E^n \longrightarrow F$ is said to be orthosymmetric if $T(x_1, ..., x_n) = 0$ whenever $x_1, ..., x_n \in E$ satisfy $x_i \perp x_j$ for some $i \neq j$..
- A bilinear map $T : E \times E \to F$ is positive if $T(x, y) \geq 0$ whenever $(x, y) \in E^+ \times E^+$, and is order bounded if given $(x, y) \in E^+ \times E^+$ there exists $a \in F^+$ such that $|T(z, w)| \leq a$ for all $(0, 0) \leq (z, w) \leq (x, y) \in E \times E$
- $T : E \times E \to F$ is (r-u) continuous if $x_n, y_n \longrightarrow 0$ (r-u) in E implies that $T(x_n, y_n) \longrightarrow 0$ (r-u) in F.

orthosymmetric multilinear mapping

- A multilinear mapping $T : E^n \longrightarrow F$ is said to be orthosymmetric if $T(x_1, ..., x_n) = 0$ whenever $x_1, ..., x_n \in E$ satisfy $x_i \perp x_j$ for some $i \neq j$..
- A bilinear map $T : E \times E \rightarrow F$ is positive if $T(x, y) \geq 0$ whenever $(x, y) \in E^+ \times E^+$, and is order bounded if given $(x, y) \in E^+ \times E^+$ there exists $a \in F^+$ such that $|T(z, w)| \leq a$ for all $(0, 0) \leq (z, w) \leq (x, y) \in E \times E$
- $T : E \times E \rightarrow F$ is (r-u) continuous if $x_n, y_n \longrightarrow 0$ (r-u) in E implies that $T(x_n, y_n) \longrightarrow 0$ (r-u) in F.

- The set $\mathcal{L}_b(E)$ of all order bounded operators on E is an ordered vector space with respect to the pointwise operations and order. The positive cone of $\mathcal{L}_b(E)$ is the subset of all positive operators.
- An element T in $\mathcal{L}_b(E)$ is referred to as an orthomorphism if, for all $x, y \in E$, $|T(x)| \wedge |y| = 0$ whenever $|x| \wedge |y| = 0$. Under the ordering and operations inherited from $\mathcal{L}_b(E)$, the set $Orth(E)$ of all orthomorphisms on E is an Archimedean Riesz space.
- The Riesz algebra E is said to be an f-algebra whenever $x \wedge y = 0$ then $xz \wedge y = zx \wedge y = 0$ for all $z \in E^+$.
- If E is a Riesz space then the Riesz space $Orth(E)$ is an f-algebra with respect to the composition as multiplication. Moreover the identity map on E is the multiplicative unit of $Orth(E)$. In particular, the f-algebra $Orth(E)$ is semiprime and commutative.
- If E is an f-algebra with unit element, then the mapping $\pi : x \longrightarrow \pi_x$ from E into $Orth(E)$ is a Riesz and algebra isomorphism, where $\pi_x(y) = xy$ for all $y \in E$.

- The set $\mathcal{L}_b(E)$ of all order bounded operators on E is an ordered vector space with respect to the pointwise operations and order. The positive cone of $\mathcal{L}_b(E)$ is the subset of all positive operators.
- An element T in $\mathcal{L}_b(E)$ is referred to as an orthomorphism if, for all $x, y \in E$, $|T(x)| \wedge |y| = 0$ whenever $|x| \wedge |y| = 0$. Under the ordering and operations inherited from $\mathcal{L}_b(E)$, the set $Orth(E)$ of all orthomorphisms on E is an Archimedean Riesz space.
- The Riesz algebra E is said to be an f-algebra whenever $x \wedge y = 0$ then $xz \wedge y = zx \wedge y = 0$ for all $z \in E^+$.
- If E is a Riesz space then the Riesz space $Orth(E)$ is an f-algebra with respect to the composition as multiplication. Moreover the identity map on E is the multiplicative unit of $Orth(E)$. In particular, the f-algebra $Orth(E)$ is semiprime and commutative.
- If E is an f-algebra with unit element, then the mapping $\pi : x \longrightarrow \pi_x$ from E into $Orth(E)$ is a Riesz and algebra isomorphism, where $\pi_x(y) = xy$ for all $y \in E$.

- The set $\mathcal{L}_b(E)$ of all order bounded operators on E is an ordered vector space with respect to the pointwise operations and order. The positive cone of $\mathcal{L}_b(E)$ is the subset of all positive operators.
- An element T in $\mathcal{L}_b(E)$ is referred to as an orthomorphism if, for all $x, y \in E$, $|T(x)| \wedge |y| = 0$ whenever $|x| \wedge |y| = 0$. Under the ordering and operations inherited from $\mathcal{L}_b(E)$, the set $Orth(E)$ of all orthomorphisms on E is an Archimedean Riesz space.
- The Riesz algebra E is said to be an f-algebra whenever $x \wedge y = 0$ then $xz \wedge y = zx \wedge y = 0$ for all $z \in E^+$.
- If E is a Riesz space then the Riesz space $Orth(E)$ is an f-algebra with respect to the composition as multiplication. Moreover the identity map on E is the multiplicative unit of $Orth(E)$. In particular, the f-algebra $Orth(E)$ is semiprime and commutative.
- If E is an f-algebra with unit element, then the mapping $\pi : x \longrightarrow \pi_x$ from E into $Orth(E)$ is a Riesz and algebra isomorphism, where $\pi_x(y) = xy$ for all $y \in E$.

- The set $\mathcal{L}_b(E)$ of all order bounded operators on E is an ordered vector space with respect to the pointwise operations and order. The positive cone of $\mathcal{L}_b(E)$ is the subset of all positive operators.
- An element T in $\mathcal{L}_b(E)$ is referred to as an orthomorphism if, for all $x, y \in E$, $|T(x)| \wedge |y| = 0$ whenever $|x| \wedge |y| = 0$. Under the ordering and operations inherited from $\mathcal{L}_b(E)$, the set $Orth(E)$ of all orthomorphisms on E is an Archimedean Riesz space.
- The Riesz algebra E is said to be an f-algebra whenever $x \wedge y = 0$ then $xz \wedge y = zx \wedge y = 0$ for all $z \in E^+$.
- If E is a Riesz space then the Riesz space $Orth(E)$ is an f-algebra with respect to the composition as multiplication. Moreover the identity map on E is the multiplicative unit of $Orth(E)$. In particular, the f-algebra $Orth(E)$ is semiprime and commutative.
- If E is an f-algebra with unit element, then the mapping $\pi : x \longrightarrow \pi_x$ from E into $Orth(E)$ is a Riesz and algebra isomorphism, where $\pi_x(y) = xy$ for all $y \in E$.

- The set $\mathcal{L}_b(E)$ of all order bounded operators on E is an ordered vector space with respect to the pointwise operations and order. The positive cone of $\mathcal{L}_b(E)$ is the subset of all positive operators.
- An element T in $\mathcal{L}_b(E)$ is referred to as an orthomorphism if, for all $x, y \in E$, $|T(x)| \wedge |y| = 0$ whenever $|x| \wedge |y| = 0$. Under the ordering and operations inherited from $\mathcal{L}_b(E)$, the set $Orth(E)$ of all orthomorphisms on E is an Archimedean Riesz space.
- The Riesz algebra E is said to be an f-algebra whenever $x \wedge y = 0$ then $xz \wedge y = zx \wedge y = 0$ for all $z \in E^+$.
- If E is a Riesz space then the Riesz space $Orth(E)$ is an f-algebra with respect to the composition as multiplication. Moreover the identity map on E is the multiplicative unit of $Orth(E)$. In particular, the f-algebra $Orth(E)$ is semiprime and commutative.
- If E is an f-algebra with unit element, then the mapping $\pi : x \longrightarrow \pi_x$ from E into $Orth(E)$ is a Riesz and algebra isomorphism, where $\pi_x(y) = xy$ for all $y \in E$.

- A Dedekind complete Riesz space E is said to be *universally complete* whenever every set of pairwise disjoint positive elements has a supremum.
- Every Archimedean Riesz space E has a unique (up to a Riesz isomorphism) universally completion denoted E^u, ie., there exists a unique universally complete Riesz space such that E can be identified with an order dense Riesz subspace of E^u.
- E^u is furnished with a multiplication, under which E^u is an f-algebra with unit element.

- A Dedekind complete Riesz space E is said to be *universally complete* whenever every set of pairwise disjoint positive elements has a supremum.
- Every Archimedean Riesz space E has a unique (up to a Riesz isomorphism) universally completion denoted E^u, ie., there exists a unique universally complete Riesz space such that E can be identified with an order dense Riesz subspace of E^u.
- E^u is furnished with a multiplication, under which E^u is an f-algebra with unit element.

- A Dedekind complete Riesz space E is said to be *universally complete* whenever every set of pairwise disjoint positive elements has a supremum.
- Every Archimedean Riesz space E has a unique (up to a Riesz isomorphism) universally completion denoted E^u, ie., there exists a unique universally complete Riesz space such that E can be identified with an order dense Riesz subspace of E^u.
- E^u is furnished with a multiplication, under which E^u is an f-algebra with unit element.

Theorem

Let E be a relatively uniformly complete Riesz spaces, F be a Hausdorff t.v.s. (not necessarily a Riesz spaces) and let $\varphi : E \times E \to F$ be a continuous orthosymmetric bilinear map then φ is symmetric

Theorem

Let E be a Riesz space, F be a Hausdorff t.v.s., and let $T : E^n \to F$ be a continuous orthosymmetric multilinear map such that $(T(E^n))'$ separates points. If $\sigma \in S(n)$ is a permutation then

$$T(x_1, ..., x_n) = T(x_{\sigma(1)}, ..., x_{\sigma(n)})$$

for all $x_1, ..., x_n \in E$.

Theorem

Let E be a relatively uniformly complete Riesz spaces, F be a Hausdorff t.v.s. (not necessarily a Riesz spaces) and let $\varphi : E \times E \to F$ be a continuous orthosymmetric bilinear map then φ is symmetric

Theorem

Let E be a Riesz space, F be a Hausdorff t.v.s., and let $T : E^n \to F$ be a continuous orthosymmetric multilinear map such that $(T(E^n))'$ separates points. If $\sigma \in S(n)$ is a permutation then

$$T(x_1, ..., x_n) = T(x_{\sigma(1)}, ..., x_{\sigma(n)})$$

for all $x_1, ..., x_n \in E$.

Let E be a Riesz space, F be a Hausdorff t.v.s., and let $T : E^n \longrightarrow F$ be a continuous orthosymmetric multilinear map such that $(T(E^n))'$ separates points . Then

$$T(\pi_1(x_1), ..., \pi_n(x_n)) = T(x_1, ..., \pi_1 ... \pi_n(x_n))$$

for all $x_1, ..., x_n \in E$ and $\pi_1, ..., \pi_n \in Orth(E)$.

orthogonally additive homogeneous polynomials

- Let E be a vector lattice and let F be a topological space. A map $P : E \to F$ is called a homogeneous polynomial of degree n (or a n-homogeneous polynomial) if $P(x) = \psi(x, .., x)$, where ψ is a n-multilinear map from E^n into F.
- A homogeneous polynomial, of degree n, $P : E \to F$ is said to be orthogonally additive if $P(x + y) = P(x) + P(y)$ where $x, y \in E$ are orthogonally (i.e. $|x| \wedge |y| = 0$).
- We denote by $\mathcal{P}_0(^nE, F)$ the set of n-homogeneous orthogonally additive polynomials from E to F.

orthogonally additive homogeneous polynomials

- Let E be a vector lattice and let F be a topological space. A map $P : E \longrightarrow F$ is called a homogeneous polynomial of degree n (or a n-homogeneous polynomial) if $P(x) = \psi(x, .., x)$, where ψ is a n-multilinear map from E^n into F.
- A homogeneous polynomial, of degree n, $P : E \longrightarrow F$ is said to be orthogonally additive if $P(x + y) = P(x) + P(y)$ where $x, y \in E$ are orthogonally (i.e. $|x| \wedge |y| = 0$).
- We denote by $\mathcal{P}_0(^nE, F)$ the set of n-homogeneous orthogonally additive polynomials from E to F.

orthogonally additive homogeneous polynomials

- Let E be a vector lattice and let F be a topological space. A map $P : E \longrightarrow F$ is called a homogeneous polynomial of degree n (or a n-homogeneous polynomial) if $P(x) = \psi(x, .., x)$, where ψ is a n-multilinear map from E^n into F.
- A homogeneous polynomial, of degree n, $P : E \longrightarrow F$ is said to be orthogonally additive if $P(x + y) = P(x) + P(y)$ where $x, y \in E$ are orthogonally (i.e. $|x| \wedge |y| = 0$).
- We denote by $\mathcal{P}_0(^nE, F)$ the set of n-homogeneous orthogonally additive polynomials from E to F.

- **Sundaresan :** Every n-homogeneous orthogonally additive polynomial $P : L^p \to \mathbb{R}$ is determined by some $g \in L^{\frac{p}{p-n}}$ via the formula $P(f) = \int f^n g d_\mu$, for all $f \in L^p$.
- **Pérez-Garcia and I. Villanueva** : Let $P : C(X) \to\to Y$ be an orthogonally additive n-homogeneous polynomial and let $\Psi : (C(X))^n \to Y$ be its unique associated symmetric multilinear operator. Then there exists a linear operator $S : C(X) \to Y$ such that $\|S\| = \|\Psi\|$ and there exists a finitely additive measure $\mu : \sum \to Y^{**}$ such that for every $f \in C(X)$, we have $P(f) = S(f^n) = \int_X f^n g d\mu$. Here, $\sum$ is the Borel σ-algebra on X.
- **Benyamini , Lassalle and Llavona** : Proven the analogous result of K. Sundaresan for the classes of Banach lattices of functions and Köthe Banach lattices.

- **Sundaresan :** Every n-homogeneous orthogonally additive polynomial $P : L^p \to \mathbb{R}$ is determined by some $g \in L^{\frac{p}{p-n}}$ via the formula $P(f) = \int f^n g d_\mu$, for all $f \in L^p$.
- **Pérez-Garcia and I. Villanueva** : Let $P : C(X) \to\to Y$ be an orthogonally additive n-homogeneous polynomial and let $\Psi : (C(X))^n \to Y$ be its unique associated symmetric multilinear operator. Then there exists a linear operator $S : C(X) \to Y$ such that $\|S\| = \|\Psi\|$ and there exists a finitely additive measure $\mu : \sum \to Y^{**}$ such that for every $f \in C(X)$, we have $P(f) = S(f^n) = \int_X f^n g d\mu$. Here, $\sum$ is the Borel σ-algebra on X.
- **Benyamini , Lassalle and Llavona** : Proven the analogous result of K. Sundaresan for the classes of Banach lattices of functions and Köthe Banach lattices.

Let E be an Archimedean vector lattice, F be a Hausdorff topological vector space (not necessarily a vector lattice), $T : E^n \longrightarrow F$ be a (ru)-continuous orthosymmetric multilinear map such that $T(E^n)'$ separates points. Then there exists a linear operator $S : \underset{i=1}{\overset{n}{\pi}} E \longrightarrow F$ such that

$$\psi(x_1, .., x_n) = S(x_1..x_n).$$

Let E be an Archimedean vector lattice, F be a Hausdorff topological vector space (not necessarily a vector lattice), $T : E^n \longrightarrow F$ be a (ru)-continuous orthosymmetric multilinear map such that $T(E^n)'$ separates points. Then there exists a linear operator $S : \underset{i=1}{\overset{n}{\pi}} E \longrightarrow F$ such that

$$\psi(x_1, .., x_n) = S(x_1..x_n).$$

Let E be an Archimedean vector lattice, F be a Hausdorff topological vector space (not necessarily a vector lattice) and let $P \in \mathcal{P}_0({}^nE, F)$ whose associated symmetric multilinear map T satisfies $T(E^n)'$ separates points. Then T is orthosymmetric.

Structure Problem

Let E be an Archimedean vector lattice, F be a Hausdorff topological vector space (not necessarily a vector lattice) and let $P \in \mathcal{P}_0({}^nE, F)$ whose associated symmetric multilinear map T satisfies $T(E^n)'$ separates points. Then there exists a linear operator $S : \underset{i=1}{\overset{n}{\pi}} E \to F$ such that

$$P(x) = S(x^n).$$

Structure Problem

Let E be an Archimedean vector lattice, F be a Hausdorff topological vector space (not necessarily a vector lattice) and let $P \in \mathcal{P}_0(^nE, F)$ whose associated symmetric multilinear map T satisfies $T(E^n)'$ separates points.Then there exists a linear operator $S : \underset{i=1}{\overset{n}{\pi}} E \to F$ such that

$$P(x) = S(x^n).$$

Our approach fails for the non continuous case

Structure Problem

Let E be the Riesz space of all real valued functions f on $[0,1]$ satisfying that there is a finite subset $(x_i)_{1\leq i\leq n}$ such that $0 = x_0 < x_1 < .. < x_n = 1$ and on each interval $[x_{i-1}, x_i)$ $f(x) = m_i(f)x + b_i(f)$ and $T(f,g) = m_0(f)b_0(g)$.

Our approach fails for the non continuous case

Structure Problem

Let E be the Riesz space of all real valued functions f on $[0,1]$ satisfying that there is a finite subset $(x_i)_{1\leq i\leq n}$ such that $0 = x_0 < x_1 < .. < x_n = 1$ and on each interval $[x_{i-1}, x_i)$ $f(x) = m_i(f)x + b_i(f)$ and $T(f,g) = m_0(f)b_0(g)$.

THANK YOU FOR YOUR ATTENTION