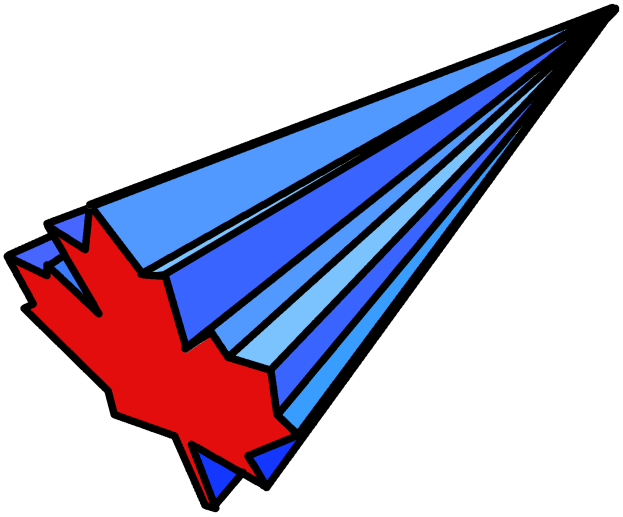
Positivity IX

July 17-21, 2017, University of Alberta, Edmonton, Canada



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Abstracts

Algebras of disjointness preserving operators

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Let A be an algebra of disjointness preserving operators on a Banach lattice X. This talk intends to give a systematic study of such algebra. A particular attention is paid to the set of all nilpotent operators in A as well as to its commutativity.

On the adjoints of some operators on Banach lattices

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Contrarily to the class of compact operators and the class of weakly compact operators, all the classes of operators defined only on Banach lattices, as AM-compact operators (resp. semi-compact operators, b-weakly compact operators, Almost Dunford-Pettis operators, order weakly compact operators), have a shortcoming.

In fact, these classes of operators do not satisfy the duality property, i.e.

1- there exist AM-compact operators (resp. semi-compact operators, b-weakly compact operators, Almost Dunford-Pettis operators, order weakly compact operators) whose adjoints are not AM-compact (resp. semi-compact operators, b-weakly compact operators, Almost Dunford-Pettis operators, order weakly compact operators), and

2- conversely, there exist operators which are not AM-compact (resp. semi-compact operators, b-weakly compact operators, Almost Dunford-Pettis operators, order weakly compact operators) but their adjoints are AM-compact (resp. semi-compact operators, b-weakly compact operators, Almost Dunford-Pettis operators, order weakly compact operators)

This problem is studied for AM-compact operators and semi-compact operators by Zaanen in his book ["*RieszSpacesII*]". In this talk, we give some necessary and sufficient conditions on the duality property of these classes of operators.

L^p-spaces with respect to conditinal expectations on Riesz spaces

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Recently, Labuschagne and Watson introduced the space $L^2(T)$ for a conditional expectation T with natural domain a Riesz space $L^1(T)$. We carry on with this process by introducing and investigae the spaces $L^p(T)$ with $p \in [1, \infty]$. This talk is based on: Y. Azouzi, M. Trabelsi, L^p-spaces with respect to conditional expectation on Riesz spaces, *J. Math. Anal. Appl.*, 447(2017), 798–816.

States on different ordered structures

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We work in set-theory without the Axiom of choice **ZF**.

We present several statements about the existence of *states* or *pure states* in various ordered groups or real ordered vector spaces with a positive order unit. We study the links between these statements and weak forms of the Axiom of Choice such as **T2** : "The product of a family of compact Hausdorff spaces is compact", or Hahn-Banach axiom **HB**.

Automatic regularity of algebra homomorphisms

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It is known that every continuous linear map from an L-space into a dual Banach lattice is regular, and so, it is natural to seek for conditions on the domain and/or codomain of a linear map under which continuity implies regularity. If the domain and codomain are Banach lattice algebras it is natural to pay special attention to those linear maps which are also algebra homomorphisms. Old results on the structure of continuous algebra homomorphisms between algebras of continuous functions provide simple examples of situations in which continuity implies regularity. In this talk I shall report on some recent positive results on automatic regularity of continuous algebra homomorphisms in the setting of Banach lattice algebras of regular operators.

New Examples of Non-reflexive Grothendieck Spaces

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For Banach lattices E and X, let $E\hat{\otimes}_{|\pi|}X$ denote the Fremlin projective tensor product of E and X. We show that if $1 < p < \infty$ and X is both an AM-space with an order unit and a Grothendieck space, then $\ell_p\hat{\otimes}_{|\pi|}X$ is a Grothendieck space. In particular, $\ell_p\hat{\otimes}_{|\pi|}\ell_\infty$, $\ell_\infty\hat{\otimes}_{|\pi|}T^*$, and $\ell_p\hat{\otimes}_{|\pi|}C(K)$ are Grothendieck spaces, where T^* is the original Tsirelson space and K is a compact stonean space, a compact σ-stonean space, or a F-space. This is a joint work with Yongjin Li.

Frolik Decompositions for Lattice-ordered Group

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Given a homeomorphism from a certain kind of topological space to itself, Frolik's Theorem provides a decomposition of the space into the clopen set of fixed points together with three clopen sets, each of whose images is disjoint from the original set. In this talk, we present such decompositions in the more general setting of (possibly noncommutative) lattice-ordered groups and group endomorphisms. The groups to which our result applies satisfy CFC, a weak condition on the polars which was previously introduced in the context of vector lattices by Abramovich and Kitover. Amongst other conditions, the endomorphisms are disjointness preserving in both directions but do not need to be order bounded. This is joint work with R. Redfield.

Orthogonally additive polynomials in Riesz spaces

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The study of orthogonally additive polynomials is of interest both from the algebrain point of view and also from the point of view of infinite dimensional analysis,

in particular the theory of holomorphic functions on infinite dimensional analysis. To the best of our knowledge the first mathematician interested in orthogonally additive polynomials was Sundaresan who obtained a representation theorem for polynomials on ℓ_p and on L_p. It is only recently that the class of such mappings has been getting more attention. We are thinking here about works on orthogonally additive polynomials and holomorphic functions and orthosymmetric multilinear mappings on different Banach lattices on $\mathbb{C}^*$-algebras, on uniformly complete vector lattices, on uniformly complete vector lattices with range space is separated convex bornological spaces and also on uniformly complete vector lattices taking values in Hausdorff topological vector spaces. Note that except for some approaches proofs of the aforementioned results are strongly based on the representation of these spaces as vector spaces of extended continuous functions. So they are not applicable to general vector lattices. Actually, the innovation of this work consists in making a relationship between orthogonally additive polynomials and orthosymmetric multilinear mappings acts on uniformly complete vector lattices taking values in a Hausdorff topological vector spaces by using only the notion of the tensor product in a vector lattice. This leads to a constructive proofs of results in all papers mentioned above.

The Bishop-Phelps-Bollobás property on bounded closed convex sets

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We show that the *BPBp* holds for bounded linear functionals on arbitrary bounded closed convex subsets of a real Banach space. For all finite dimensional Banach spaces X and Y the pair (X, Y) has the *BPBp* on every bounded closed convex subset D of X. we also prove that for a Banach space Y with property (β) the pair (X, Y) has the *BPBp* on every bounded closed absolutely convex subset D of an arbitrary Banach space X. For a bounded closed absorbing convex subset D of X with positive modulus convexity we get that the pair (X, Y) has the *BPBp* on D for every Banach space Y. We further obtain that for an Asplund space X and for a locally compact Hausdorff L, the pair $(X, C_0(L))$ has the *BPBp* on every bounded closed absolutely convex subset D of X.

Asymmetric norms, partially ordered normed spaces and injectivity

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An asymmetric norm on a real vector space X is a function $p : X \to \mathbb{R}^+$ such that for all $x, y \in X$, $\lambda \geq 0$, $p(x) = 0 = p(-x) \Rightarrow x = 0$, $p(\lambda x) = \lambda p(x)$ and $p(x + y) \leq p(x) + p(y)$. For such a p, $p^s(x) = \max\{p(x), p(-x)\}$ defines a norm on X and the set $C_p = \{x \in X : p(-x)\}$ is a p^s-closed normal cone in X. Using these, we can associate with every asymmetrically normed space (X, p) a partially ordered normed space (X, p^s) ordered by the cone C_p. Conversely, starting with a normed space $(X, ||\cdot||)$ partially ordered by a closed normal cone C, we can associate an asymmetric norm p on X defined by $p(x) = \inf\{||x + y|| : y \in C\}$. In this talk we explore the nature of this correspondence, with particular attention being paid to the relationhip between the notions of injectivity in the category of asymmetrically normed spaces with contracting linear maps and injectivity in the category of partially ordered normed spaces with contracting positive linear maps.

Disintegration of positive isometric group representations on L^p-spaces

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Let (X, μ) be a separable probability space, let $1 \leq p < \infty$, and let G be a Polish locally compact group acting in a strongly continuous manner on $\mathrm{L}^p(X, \mu)$ as isometric lattice automorphisms that leave the constants fixed. We shall explain how the representation of G on $\mathrm{L}^p(X, \mu)$ can be disintegrated into order indecomposable representations, using a disintegration theorem for invariant measures into ergodic ones and L^p-direct integrals of Banach spaces.

This is joint work with Jan Rozendaal.

Triangularizability of trace-class operators with increasing spectrum

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For any measurable set E of a measure space (X, μ), let P_E be the (orthogonal) projection on the Hilbert space $L^2(X, \mu)$ with the range $\operatorname{ran} P_E = \{f \in L^2(X, \mu) : f = 0 \text{ a.e. on } E^c\}$ that is called a standard subspace of $L^2(X, \mu)$. Let T be an operator on $L^2(X, \mu)$ having increasing spectrum relative to standard compressions, that is, for any measurable sets E and F with $E \subseteq F$, the spectrum of the operator $P_E T|_{\operatorname{ran} P_E}$ is contained in the spectrum of the operator $P_F T|_{\operatorname{ran} P_F}$. In 2009, Marcoux, Mastnak and Radjavi asked whether the operator T has a non-trivial invariant standard subspace. They answered this question affirmatively when either the measure space (X, μ) is discrete or the operator T has finite rank. We study this problem in the case of trace-class kernel operators.

Uτ-topologies in locally solid Riesz spaces

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Recently, various types of unbounded convergence in Riesz spaces were introduced and investigated. Some of them are topological, others are not. In the present talk we discuss conditions under which the unbounded convergence has a topological nature. Several basic facts on this (so-called unbounded) topology in locally solid Riesz spaces are stated.

A factorization for non-linear operators

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A class of non-linear positive operators on Banach lattices are characterized in terms of a factorization. Specifically, for E and F appropriate Banach lattices, it is shown that $T : E \to F$ is in this class if and only if $T = L \circ S$ where S is similar to an orthomorphism from E to E and L is a linear operator from E to F. The operators in question are orthogonally additive and satisfy a certain homogeneous condition. Other factorizations are also considered.

Trajectorial Martingales. Convergence and Integration

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Starting with a trajectory space, providing a non-stochastic analogue of a discrete time martingale process, we use the notion of superreplication to introduce definitions for null and full functions and the associated notion of a property holding almost everywhere (a.e.). The latter providing what can be seen as the worst case analogue of sets of measure zero in a stochastic setting. The a.e. notion is used to prove the pointwise convergence, on a full set of the original trajectory space, of the limit of a trajectorial transform sequence. The setting also allows to construct a natural integration operator

On weakly p-summable sequences in Banach lattices

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We consider properties on a Banach lattice E that will imply that $(|x_i|)$ is weakly p-summable when (x_i) is weakly p-summable. A subset W of E is said to be $(1,p)$-limited if for each weakly 1-summable sequence (x'_n) in E' there exists $(\lambda_n) \in \ell_p$ so that $|\langle x'_n, x\rangle| \leq \lambda_n$ for all $x \in W$ and all $n \in \mathbb{N}$. We show that each disjoint sequence in the solid hull of a $(1,p)$-limited set is weakly p-summable. This result is then used to discuss domination properties of some classes of operators (such as the so called p-convergent and weak* p-convergent operators) on Banach lattices as well as some applications to geometrical properties of Banach lattices (such as the Schur property of order p). For instance, if E is a Banach lattice with type q (with $1 < q \leq 2$) and $p \geq q'$, then relatively weakly compact sets in E are $(1,p)$-limited.

When are order closures of sublattices order closed?

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Let Y be a sublattice of a vector lattice X. We consider the problem of identifying the smallest order closed sublattice of X containing Y. It is known that the analogy with topological closure fails. Let $\overline{Y}^o$ be the order closure of Y consisting of all order limits of nets of elements from Y. Then $\overline{Y}^o$ need not be order closed. We show that in many cases the smallest order closed sublattice containing Y is in fact the second order closure $\overline{\overline{Y}^o}^o$ and the uo-closure $\overline{Y}^{uo}$. We also show that if X is a σ-order complete Banach lattice, then the condition that $\overline{Y}^o$ is order closed for every sublattice Y with two positive generators characterizes order continuity of the norm of X. However, if one of the two positive generators of Y lies in the order continuous part of X, it is shown that $\overline{Y}^o$ is always order closed. We may also connect these results with applications to a fundamental result in financial economics concerning the spanning power of options written on a financial asset.

This talk is based on joint work with Denny H. Leung.

The talk is dedicated to Professor Ioannis A. Polyrakis on the occasion of his retirement.

Higher order Jensen-convex functionals that are

measurable on curves

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I. Labuda and R. D. Mauldin [2] have solved in affirmative the following S. Mazur's problem posed about 1935 (see [3]):

> *"In a space E of type (B), there is given an additive functional $F(x)$ with the following property: If $x(t)$ is a continuous function in $0 \leq t \leq 1$ with values in E, then $F(x(t))$ is a measurable function. Is $F(x)$ continuous ?"*

In [1] we have shown that the same remains true in the case where F is a Jensen-convex functional on an open and convex subdomain of a real Banach space. Now, we shall study the possibilities of an extension of this result to Jensen-convexity of higher orders.

References

[1] R. Ger, *Mazur's criterion for continuity of convex functionals*, Bull. Polish Acad. Sci., Mathematics **43** (1995), 263-268.

[2] I. Labuda & R. D. Mauldin, *Problem 24 of the "Scottish Book" concerning additive functionals*, Colloquium Math. **48** (1984), 89-91.

[3] *The Scottish Book, Mathematics from the Scottish Café*, Edited by R. Daniel Mauldin, Birkhäuser, Boston-Basel-Stuttgart, 1981.

Convergence of positive operator semigroups

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In this joint work with Jochen Glück we give new conditions for strong convergence of positive operator semigroups as time tends to infinity. This is achieved by a new approach that combines the splitting theorem by Jacobs, de Leeuw and Glicksberg with a purely algebraic result about positive group representations. Thus, we obtain convergence theorems not only for one-parameter semigroups but also for a much larger class of semigroup representations without any continuity or regularity assumption in time. In particular, this generalizes results from the literature that, under technical assumptions, a bounded positive strongly continuous semigroup that contains or dominates a kernel operator converges strongly as time tends to infinity.

Eventual Positivity of Operator Semigroups

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During the last two decades, a deep and comprehensive theory of eventually positive matrices has been developed. A matrix $T \in \mathbb{R}^{d\times d}$ is called *eventually positive* if the entries of T^n are positive for all sufficiently large n. Similarly, one calls a one-parameter matrix semigroup $(e^{tA})_{t\geq 0}$, with generator $A \in \mathbb{R}^{d\times d}$, *eventually positive* if the entries of e^{tA} are positive for all sufficiently large times t.

While the classical theory of positive matrices was generalised to operators on infinite dimensional spaces several decades ago, it was only recently when the need became apparent to study eventually positive operators and semigroups in infinite dimensions, too. As it turned out, eventually positive behaviour occurs in several evolution equations associated with concrete differential operators such as the bi-Laplace operator and the Dirichlet-to-Neumann operator. This suggests that the development of a general theory of eventual positivity in infinite dimensions is a worthwhile objective.

In this talk we present several corner stones of a theory of eventual positivity on infinite dimensional Banach lattices, both for powers of single operators T and for one-parameter semigroups $(e^{tA})_{t\geq 0}$. Our focus is, on the one hand, on spectral theoretical results which yield strong and surprising generalisations of classical Perron–Frobenius and Kreĭn–Rutman type theorems. On the other hand, we discuss ways to *characterise* eventual positivity by means of spectral properties – an approach which is most useful if we want to prove eventual positivity for the solutions of a concrete evolution equation.

The talk is partially based on joint work with Daniel Daners (University of Sydney, Australia) and James B. Kennedy (University of Lisbon, Portugal).

Probability Theory and Vector Lattices: A characterization of Brownian motion

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In the first part of this talk a dictionary, relating concepts in the theories of Probability and Vector Lattices, will be established and some of the main results will be mentioned. An abstract definition of Brownian motion will be given in the second part and a discussion, including Itô's formula and Lévy's theorem, will lead to a characterization of Brownian motion.

A geometric inequality on the poisive cone and an application

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Let H be a Hilbert space. Suppose that $\|\cdot\|$ is a complete unitarily invariant uniform norm on $B(H)$. Then the inequality

$$\|\log(a^{\frac{1}{2}}ba^{\frac{1}{2}})\| \leq \|\log a\| + \|\log b\| \tag{1}$$

holds for every pair $a, b \in B(H)_+^{-1}$. Applying this inequality we prove that the positive cone of a unital C^*-algebra is a *generalized gyrovector space* with respect to these norms. As an application we exhibit the form of surjective isometries on the poisitve cones with respect to the metric induced by these norms.

Strictly singular operators on $L^p - L^q$ spaces and interpolation

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The classes $V_{p,q}$ of strictly singular non-compact operators between $L_p - L_q$ spaces $(1 \leq p\,,\, q < \infty)$ are studied. We identify the points $(\frac{1}{p}, \frac{1}{q})$ for which an

operator $T \in V_{p,q}$ as a part of the boundary of the characteristic set $L(T)$ of the operator T (in Krasnoselskii and Zabreiko's sense). This allow to obtain interpolation results for strictly singular operators between $L_p - L_q$ spaces. In particular we extend Krasnoselskii compact interpolation theorem to the strictly singular setting. The special case of regular $V_{p,q}$ operators is also studied. The talk is based on a joint work with E.M.Semenov and P.Tradacete.

Cauchy quotient means and their properties

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Means belonging to the class of beta-type functions, i.e. which can be written in the form

$$B_{f,k}(x_1, \ldots, x_k) = \frac{f(x_1) \cdot \ldots \cdot f(x_k)}{f(x_1 + \ldots + x_k)},$$

for some positive function f, are examined.

Moreover, the three other classes of Cauchy quotients, each of them naturally linked to one of the remaining Cauchy equations, is introduced and some of their properties are discussed.

Joint work with Janusz Matkowski.

1. M. Himmel, J. Matkowski, *Homogeneous beta-type functions (J. Class. Anal., Volume 10, Number 1 (2017), 59–66.).*

2. M. Himmel, J. Matkowski, *Beta-type means (to appear in J. Difference Equ. Appl.)*

Positive representations of $\mathrm{C}_0(X)$

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If X is a locally compact Hausdorff space, then a representation of the complex $\mathbb{C}^*$-algebra $\mathrm{C}_0(X)$ on a Hilbert space H is given by a spectral measure. This measure takes its values in the orthogonal projections on H. It is a natural question to ask whether something similar is true for a positive representation of the ordered Banach algebra $\mathrm{C}_0(X)$ on a Banach lattice E. If E is a KB-space, then the answer is affirmative: the representation is given by a spectral measure that takes its values in the positive projections on X; see [1].

The proofs in [1] make use of the fact that E is a Banach space, but there are some results in [1] suggesting that a purely order-theoretic more general approach might also be possible. In this lecture, we shall explain that this is indeed the case.

As a preparation, we shall sketch an integration theory for measures taking values in a suitable partially ordered vector space E. After that, we shall discuss a Riesz representation theorem for a positive map $T\colon \mathrm{C}_0(X) \to E$. Under mild conditions, this is given by a positive E-valued measure. In the next step, we apply the previous result to a positive representation $\pi\colon \mathrm{C}_0(X) \to A$, where A is a suitable partially ordered algebra. In that case, the pertinent positive A-valued measure turns out to take values in the idempotents of A. If A equals the regular operators on a suitable partially ordered vector space E, then the previous result yields a spectral measure for π that takes its values in the positive projections on E.

References

[1] M. de Jeu, F. Ruoff, *Positive representations of* $\mathrm{C}_0(X)$*. I*, Ann. Funct. Anal. **7** (2016), 180–205.

Approximation properties of a Banach space and its subspaces

Johnson, Bill (johnson@math.tamu.edu)
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1. HAPpy Banach spaces.
2. Joint AP for a Banach space X and its subspace Y.
3. Erdos meets Lidskii.

Unbounded norm topology

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In a normed vector lattice X the usual norm convergence induces the so-called *unbounded norm convergence* or *un-convergence* in the following way: we say that a net (x_α) un-converges to $x \in X$ if $\||x_\alpha - x| \wedge u\| \to 0$ for each $u \in X_+$. If $X = L_1(\mu)$ for some finite measure space then un-convergence agrees with the convergence in measure, and if $X = C(K)$ for some compact Hausdorff space K, then un-convergence agrees with uniform convergence. In this talk we discuss properties of *unbounded norm topology on* X induced by un-convergence. In particular, we consider metrizability, compactness, completeness, etc. We also provide a natural extension of unbounded norm topology from X to its universal completion X^u.

Convergence theorems of nonlinear integral functionals

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The Lebesgue integral is used for aggregating an infinite number of inputs into a single output value and is continuous with respect to inputs by the Lebesgue convergence theorem. This continuity is a guarantee for certain robustness and consistency and a non chaotic behavior in the aggregation process.

The Choquet integral $\mathrm{Ch}(\mu, f)$ and the Sugeno integral $\mathrm{Su}(\mu, f)$ for a measurable function $f\colon X \to [0, \infty]$ and a nonadditive (also called monotone) measure $\mu\colon \mathcal{A} \to [0, \infty]$ on a measurable space $(X, \mathcal{A})$ may be considered as nonlinear aggregation functions with input f. For those functions their continuity corresponds to the convergence theorem of integrals, which means that the limit of the integrals of a sequence of functions is the integral of the limit function. Thus many attempts have been made to formulate the monotone, the bounded, and the dominated convergence theorem for nonlinear integrals such as the Choquet, the Šipoš, the Sugeno, and the Shilkret. However, to the best of knowledge, there is no unified approach to such convergence theorems in literature that are simultaneously applicable to both the Lebesgue integral as a linear integral and the Choquet, the Šipoš, the Sugeno, and the Shilkret integrals as nonlinear integrals. The purpose of the talk is to present a unified approach to convergence theorems for such linear and nonlinear integrals.

A nonlinear integral may be viewed as a nonlinear functional $I\colon \mathcal{M}(X)\times\mathcal{F}^+(X)\to[0,\infty]$, where $\mathcal{M}(X)$ is the set of all nonadditive measures $\mu\colon\mathcal{A}\to[0,\infty]$ and $\mathcal{F}^+(X)$ is the set of all $\mathcal{A}$-measurable functions $f\colon X\to[0,\infty]$. So we formulate a general type of convergence theorem for such a functional. In particular we announce that the monotone, the bounded, and the dominated convergence theorem of nonlinear integrals follow from our "one-size-fits-all" convergence theorems of functionals regardless of the type of nonlinear integrals. A key ingredient is a perturbation of functional that manages not only the monotonicity of a functional I but also the small change of the functional value $I(\mu,f)$ arising as a result of adding small amounts to a measure μ and a function f in the domain of I.

Topological Connectedness and Behavioral Assumptions on Preferences: A Two-Way Relationship

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This talk offers a synthetic treatment of the relationship between assumptions on a binary relation and those on the choice space over which the binary relation is defined. This pursuit of the Eilenberg-Sonnenschein research program in an exclusively topological register hinges on the centrality of connectedness for results on the necessity and the sufficiency for transitivity and/or completeness of the binary relation. The characterizations of topological connectedness that our work offers is novel to both mathematical and economic literatures, and also furnishes generalizations of results due to Eilenberg, Sonnenschein, Schmeidler and their followers that connect topological and order structures.

This is a joint work with Metin Uyanık.

Bishop-Phelps-Bollobás property for bilinear forms

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In 1963, Lindenstrauss studied possible extensions of the Bishop-Phelps theorem to the vector-valued case, and this is the starting point of research on norm attaining operators. Similarly, in 2008, Acosta, Aron, García and Maestre started the study of vector-valued versions of the Bishop-Phelps-Bollobás theorem with introducing the so-called Bishop-Phelps-Bollobás property. In this talk, we introduce the Bishop-Phelps-Bollobás property for bilinear forms with both positive result and negative one.

The dual Radon-Nikodym property in Banach $C(K)$-modules of finite multiplicity.

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We extend the well-known criterion of Lotz for the dual Radon-Nikodym property (RNP) in the class of Banach lattices to the classes of finitely generated Banach $C(K)$-modules and Banach $C(K)$-modules of finite multiplicity. Namely, we prove that if X is a Banach space from one of these classes then its Banach dual $X^\star$ has the RNP iff X does not contain a closed subspace isomorphic to ℓ^1.

The Itô integral for Brownian motion in a vector lattice

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A construction and properties of the Itô integral for Brownion motion in vector lattices will be discussed.

On Characterizing the spectra of nonnegative matrices

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The nonnegative inverse eigenvalue problem (NIEP) asks for necessary and sufficient conditions on a list

.

$$\sigma := (\lambda_1, \quad ... \quad , \lambda_n)$$

of complex numbers, in order that σ be the list of eigenvalues of an (entry-wise) nonnegative real $n \times n$ matrix.

If σ is the spectrum of a nonnegative matrix A, we say that σ is **realizable** and refer to A as a **realizing** matrix**.**

We will discuss the current status of knowledge on this problem and the related problems (SNIEP), where one seeks a realizing matrix that is symmetric, and (DNIEP), where one seeks a realizing matrix that is diagonalizable, in addition to being nonnegative.

We will also present some recent results, obtained in collaboration wirh R. Loewy and H. Šmigoc (Math.Ann. **364** (2016) 687-707) on sign patterns of certain Taylor series associated with nonnegative matrices.

M-operators on Partially Ordered Banach Spaces

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For a matrix $A \in \mathbb{R}^{n\times n}$ there are several well-known properties that are equivalent to A being an invertible M-matrix. One of them is the positive stability of A. Generalizations of this property to partially ordered Banach spaces are considered and linked with some other equivalent conditions. An important theorem on singular irreducible M–matrices is generalized to partially ordered Banach spaces using the concept of M-operators and irreducible operators. Certain other invertibility conditions of M-operators are also investigated.

This is based on a joint work with PD Dr. Anke Kalauch (TU Dresden, Germany) and Prof Dr. K. C. Sivakumar (IIT Madras, India).

A non-linear Bishop-Phelps-Bollobás type theorem

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A Bishop-Phelps-Bollobás type theorem is introduced on the unital uniform algebra $\mathcal{A}_{w^*u}(B_{X^*})$. $\mathcal{A}_{w^*u}(B_{X^*})$ consists of all w^*-uniformly continuous functions on the closed unit ball B_{X^*} which are holomorphic on the interior of B_{X^*}. We show that this

result holds for $\mathcal{A}_{w^*u}(B_{X^*})$ if X^* is uniformly convex or X^* is the uniformly complex convex dual space of an order continuous absolute normed space. The vector-valued case is also studied.

Order and topology of convex sets in Orlicz spaces, with applications to risk measures

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Let S be a subset of an Orlicz space L^Φ. The *order closure* $\overline{S}^o$ of S consists of all $f \in L^\Phi$ which is the order limit of an order bounded net in S. (Equivalently, the almost everywhere limit of an order bounded sequence in S.) S is said to be *order closed* if $S = \overline{S}^o$. In contrast to topological closure, $\overline{S}^o$ itself need not be order closed. An important question arising from the theory of risk measures in financial mathematics asks whether the order closure of a convex set C in L^Φ coincides with its closure under a weak topology (generated by a point separating order ideal of the dual space $(L^\Phi)^*$). I will discuss solutions to this problem (depending on the convex set and the weak topology considered) and, in particular, how the analog of the Krein-Smulyan Theorem for the topology $\sigma(L^\Phi, L^\Psi)$ comes into the picture.

Joint work with Niushan Gao, Cosimo Munari and Foivos Xanthos.

Some loose ends on unbounded order convergence

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The notion of almost everywhere convergence has been generalized to vector lattices as unbounded order convergence, which proves to be a very useful tool in the theory of vector and Banach lattices. In this talk, we establish some new results on unbounded order convergence that tie up some loose ends. In particular, we show that every norm bounded positive increasing net in anorder continuous Banach lattice is uo-Cauchy and that every uo-Cauchy net in an order continuous Banach lattice has a uo-limit in the universal completion.

Galois connections between generating systems of sets and sequences

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The talk is based on the following construction due to Stephani from 1980. Given a collection of sequences in a Banach space X, we consider all the subsets in which every sequence has a subsequence from the given collection. This construction produces a collection of subsets of X. Conversely, given a collection of subsets of X, we consider all the sequences contained in those sets; this procedure gives us a collection of sequences in X. Thus, we have maps from collections of sequences to collections of subsets, and vice versa. We study these maps and various structures that arise as byproducts of these maps. We also investigate order properties of these maps.

The research was supported by institutional research funding IUT20-57 of the Estonian Ministry of Education and Research.

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Vector-valued Rubio de Francia extrapolation to Banach function spaces

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We extend Rubio de Francia's extrapolation theorem for functions valued in UMD Banach function spaces. In particular we show that if an operator T is bounded on $L^p(\mathbb{R}^d, w)$ for some $p > p_0 > 0$ and all Muckenhoupt weights $w \in A_{p/p_0}$, then T extends to a bounded operator on the weighted Bochner space $L^p(\mathbb{R}, w; X)$ for all p_0-convex Banach function spaces such that the p_0-concavification X^{p_0} has the UMD property. The special case $p_0 = 1$ and without weights in the conclusion is due to Rubio de Francia. Consequences include vector-valued Littlewood–Paley–Rubio de Francia-type estimates, $L^p(\mathbb{R}, w; X)$-boundedness of the variational Carleson operator and $L^p(\mathbb{R}, w; X)$-boundedness of operator-valued Fourier multipliers.

This is joint work with Alex Amenta and Mark Veraar (TU Delft).

Order continuous operators on pre-Riesz spaces

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In this talk we consider operators between Archimedean pre-Riesz spaces. A *pre-Riesz space* is a partially ordered space which can be order densely embedded into a vector lattice (the so-called *vector lattice cover*).

If X_1 and X_2 are Archimedean pre-Riesz spaces, then the operator spaces $L_r(X_1, X_2)$ and the directed part of $L_{oc}(X_1, X_2)$ (order continuous operators) are Archimedean pre-Riesz spaces as well. Is it realistic to hope that $L_r(X_1, X_2^\delta)$ and $L_{oc}(X_1, X_2^\delta)$ are their respective vector lattice covers? The answer is negative, even if X_1 and X_2 are vector lattices with "nice" properties.

We give some conditions under which a vector lattice cover of the directed part of $L_{oc}(X_1, X_2)$ consists of operators. This vector lattice cover even turns out to be the Dedekind completion.

Finally, we give another reason why we can not simply make the range space Dedekind complete: Even if we consider a particular operator, the properties of the operator might change. We give an example of an operator $T\colon X_1 \to X_2$ which is not order continuous, but belongs to $L_{oc}(X_1, X_2^\delta)$, i.e. is order continuous if viewed as an operator $T\colon X_1 \to X_2^\delta$.

um-Topology in Multi-Normed Vector Lattices

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Let $\mathcal{M} = \{m_\lambda\}_{\lambda\in\Lambda}$ be a separating family of lattice seminorms on a vector lattice X, then $(X, \mathcal{M})$ is called a multi-normed vector lattice (or MNVL). We write $x_\alpha \xrightarrow{\text{m}} x$ if $m_\lambda(x_\alpha - x) \to 0$ for all $\lambda \in \Lambda$. A net x_α in an MNVL $X = (X, \mathcal{M})$ is said to be unbounded m-convergent (or um-convergent) to x if $|x_\alpha - x| \wedge u \xrightarrow{\text{m}} 0$ for all

$u \in X_+$. um-Convergence generalizes un-convergence [3, 4] and uaw-convergence [5], and specializes up-convergence [1] and $u\tau$-convergence [2]. um-Convergence is always topological, whose corresponding topology is called unbounded m-topology (or um-topology). We show that, for an m-complete metrizable MNVL $(X, \mathcal{M})$, the um-topology is metrizable iff X has a countable topological orthogonal system. In terms of um-completeness, we present a characterization of MNVLs possessing both Lebesgue's and Levi's properties. Then, we characterize MNVLs possessing simultaneously the σ-Lebesgue and σ-Levi properties in terms of sequential um-completeness. Finally, we prove that any m-bounded and um-closed set is um-compact iff the space is atomic and has Lebesgue's and Levi's properties.

The talk will be presented via skype.

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Real interpolation and conditional expectation operators on ordered ideals of $L^1(0,1)$-space.

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(1) We considerably improve the well known Calderon - Ryff interpolation theorem and prove that a linear space X of Lebesgue-measurable functions on , $(0,1)$, $L^\infty \subseteq X \subseteq L^1$, is an interpolation space if and only if every conditional expectation operator maps X into itself.

(2) We introduce the notion of verifying σ-subalgebra, i.e. such a σ-subalgebra that X is an interpolation space if and only if the corresponding conditional expectation operator maps X into itself.

We completely characterize all independently complemented verifying σ-subalgebras.

(3) Let $\mathcal{N}_f$ be a principal symmetric ideal generated by $f \in L^1$. We provide an effective criterion in terms of f allowing to decide when every conditional expectation operator corresponding to an independently complemented σ-subalgebra acts on X.

The talk will be presented by A.Meckler (over a teleconference) and by A. Kitover.

Lipschitz structure in ordered Banach spaces

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In previous work by the speaker with Marcel de Jeu, very simple constructions showed how, in that specific situation, the category of Banach lattices is too small and one is forced to work in the larger category of ordered Banach spaces. Still, Banach lattices possess useful basic features which significantly simplify constructions like completions of Banach-lattice-valued function spaces. One such feature is that, in every Banach lattice, the lattice operations are Lipschitz continuous.

This motivates the following general question for ordered Banach spaces which will be the main topic of this talk:

For a real Banach space X ordered by a closed and generating cone/wedge C, do there exist **Lipschitz continuous** functions $(\cdot)^{\pm} : X \to C$ so that $x = x^+ - x^-$ for all $x \in X$?

The speaker's research is sponsored by The Claude Leon Foundation.

Applications of the scarcity theorem in ordered Banach algebras

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B. Aupetit's scarcity theorem ([1], Theorem 3.4.25) is a very deep result, which, in very general terms, states that if a function f is analytic on a domain D in the complex plane and with values in a Banach algebra, then either the subset of D on which the spectrum of f is finite is "very small" in some sense or it is the whole of D, in which case the spectrum of f is even uniformly finite on D. Several applications of the scarcity theorem have been illustrated in [1], as well as in a number of subsequent papers by other authors. We apply Aupetit's scarcity theorem to obtain stronger versions of many spectral-theoretical results in ordered Banach algebras in which the algebra cone has generating properties, such as in the algebra of all regular operators on a complex Banach lattice, where the algebra cone generates the entire algebra — see [2]. We show, for instance, that a semisimple ordered Banach algebra A with generating algebra cone is isomorphic to the algebra of all complex numbers if and only if the spectrum of each positive invertible element consists of one element only. In addition, results involving finite rank elements, the centre and the radical are obtained.

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Unbounded norm convergence in Banach lattices

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Unbounded norm convergence was originally introduced as a tool for studying measures of non-compactness of operators. In this talk I will discuss properties of

this convergence and, in particular, how it may be viewed as a generalization of convergence in measure to the setting of Banach lattices.

Large sublattices in subsets of Banach lattices

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Suppose A is a subset of a Banach lattice X. Under what conditions does A contain a (closed) infinite dimensional sublattice? We obtain a positive answer, for instance, when X is either order continuous or a $C(K)$ space, and A is a complement of $Z\backslash\{0\}$, where $Z \subset X$ is a closed subspace of infinite codimension.

Extending algebra homomorphisms to spectral measures; a Boolean algebra approach

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Let K be a compact (or locally compact) Hausdorff space. Let m be a bounded positive algebra homomorphism of $C(K)$ (or $C_0(K)$) into $L_r(E)$, the regular operators on a Banach lattice E. Let Σ denote the σ-algebra of the Borel subsets of K. A Boolean algebra homomorphism ϕ from Σ into $L_r(E)$ is called a **countably additive spectral measure** if it is countably additive with respect to the strong operator topology on $L_r(E)$. We study necessary and sufficient conditions that m must satisfy in order to have an extension to a positive bounded algebra homomorphism ϕ_m from $B(\Sigma)$ (the algebra of bounded Borel measurable functions on K) into $L_r(E)$ such that $\phi_m|_\Sigma$ is a countably additive spectral measure.

We also study the necessary and sufficient conditions that the Banach lattice E must satisfy in order to have the above extension property for every positive bounded algebra homomorphism m as above.

Burkholder inequalities in Riesz spaces

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The extent to which the Burkholder Inequalities in classical Stochastic Analysis can be generalized to the more general setting of Stochastic Analysis in Riesz spaces (i.e., vector lattices)

We recall some of the relevant ideas.

Let $\{(X_n, \mathcal{F}_n) : n \geq 1\}$ be a martingale. Martingale increments are given by

$$\Delta X_1 = X_1 \quad \text{and} \quad \Delta X_n = X_n - X_{n-1} \text{ for all } n = 2, 3, ...$$

and the Quadratic Variation Process is defined by

$$S_n(X) = (\Delta X_1)^2 + \cdots + (\Delta X_n)^2 \text{ for all } n = 1, 2, ...$$

Roughly speaking, the Burkholder inequality stipulates that, as far as L^p-norms are concerned, $S_n^{1/2}$ and X_n increase at the same rate. More precisely, for every $p \in (1, \infty)$ there do exist positive real numbers a_p and b_p such that

$$a_p \left\| S_n^{1/2} \right\|_p \leq \|X_n\|_p \leq b_p \left\| S_n^{1/2} \right\|_p .$$

Removability results for subharmonic functions, for harmonic functions and for holomorphic functions

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We begin with an improvement to Blanchet's et al. extension result for subharmonic functions. With the aid of our improvement we then give extension results both for harmonic and for holomorphic functions. Our results for holomorphic functions are related to Besicovitch's and Shiffman's well-known extension results, at least in some sense.

An order theoretical characterisation of JB-algebras

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A striking link between Jordan algebras and the geometry of cones was discovered by Koecher and Vinberg. They proved independently that a finite dimensional cone is the cone of squares in a formally real Jordan algebra if and only if it is symmetric (self-dual and homogeneous). The self-duality of the cone can not be extended to infinite dimensions, so there is no direct infinite dimensional analogue of the Koecher-Vinberg theorem. Inspired by an alternative finite dimensional order theoretic characterisation of formally real Jordan algebras by Walsh (2013), the notion of a symmetric cone is replaced by an order unit space on which there is a so called antitone map. These have natural generalisations to infinite dimensions and a first step towards the order theoretical characterisation of JB-algebras is presented for spin factors (joint work with B. Lemmens and H. van Imhoff).

Wittstock moduli of elementary operators and their application to "geneneralized notions of amenability"

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Let $\mathfrak{A}$ be a C^*-algebra, and let $T : \mathfrak{A} \to \mathcal{B}(\mathfrak{H})$ be completely bounded. We call $|T| : \mathfrak{A} \to \mathcal{B}(\mathfrak{H})$ a *Wittstock modulus* of $\mathfrak{A}$ if $|T|$ is completely positive such that $\||T|\|_{cb} \leq \|\|T\|_{cb}$ and $|T| \pm \operatorname{Re} T \geq 0$ and $|T| \pm \operatorname{Im} T \geq 0$. As a consequence of G. Wittstock's celebrated decomposition theorem, every completely bounded operator has a Wittstock modulus. We give a concrete description of Wittstock moduli of elementary operators and put it to work towards settling the question of whether a pseudo- or approximately amenable C^*-algebra is actually amenable.

Fatou's Lemma, Galerkin Approximations and the Existence of Walrasian Equilibria in Infinite Dimensions

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This essay has three objectives: (i) to report recent generalizations of Fatou's lemma to multi-functions taking values in a Banach space, and framed in terms of both Bochner and Gelfand integration; (ii) to delineate the importance of Galerkin approximations in Walrasian general equilibrium theory with a continuum of agents and commodities; and thereby (iii) to present two new results on the existence of a

Walrasian equilibrium in economies where the continuum of agents is formalized as a saturated measure space.

Joint work with Mohammed A. Khan

(Pre)-Duals of the band of integral operators

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Previously we gave two descriptions of the dual of the space $(L^{p'} \otimes L^p)^{dd}$ of regular integral operators on an L^p space. In this talk we will present the extension of these results to the space $(L^{p'} \otimes L^q)^{dd}$ for $q < p$. Then we extend the results for integral operators from a Banach function space E into a Banach function space F, where we will assume $\sigma(F) < s(E)$ (or in one of the two cases the slightly weaker assumption that E is p-convex and F is p concave for some $1 < p < \infty$).

Riesz-Kantorovich formulas for operators on multi-wedged spaces

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In this talk we build on the theory of two recent papers by M. de Jeu and M. Messerschmidt on vector spaces equipped with an arbitrary set of wedges by considering a more general notion of supremum in this setting. Indeed, if E is a vector space, $\mathcal{W}$ is a set of wedges in E, I is an index set, and $(x_i, W_i)_{i\in I} \in (E \times \mathcal{W})^I$ then any $z \in E$ that satisfies

$$\bigcap_{i\in I}(x_i + W_i) = z + \bigcap_{i\in I} W_i$$

can be viewed as a generalized supremum, which we call a multi-supremum, of $(x_i, W_i)_{i\in I}$. Multi-lattices are multi-wedged vector spaces that are closed under multi-suprema and are thus an abstraction of vector lattices. Despite losing some fundamental properties of vector lattices in the multi-lattice setting, we obtain Riesz-Kantorovich formulas for multi-suprema in certain multi-wedged spaces of operators.

Banach limits and their applications

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Let l_∞ be the space of all bounded sequence $x = (x_1, x_2, \ldots)$ with the norm $\|x\|_{l_\infty} = \sup\limits_{n\in\ \mathbb{N}} |x_n|$, and standard semiordering. A linear functional $B \in l_\infty^*$ is said to be a Banach limit if $Bx \geq 0$ for any $x \in l_\infty$, $x \geq 0$, $B(1, 1, \ldots) = 1$ and $Bx = BTx$ for any $x \in l_\infty$ where T is the translation operator.

The existence of Banach limits was established by S. Mazur and was presented in the famous S. Banach's book. Denote by $\mathfrak{B}$ the set of all Banach limits. It follows from the definitions that $Bx = \lim\limits_{n\to\infty} x_n$ for any convergent x and that $\mathfrak{B}$ is a convex closed set on the unit sphere of l_∞^*.

We shall denote by Γ the set of all operators H satisfying the following conditions: $H \geq 0$, $H(1, 1, \ldots) = 1$, $Hc_0 \subset c_0$ and $\limsup\limits_{j\to\infty}(A(I - T))_j \geq 0$ for all $x \in l_\infty$, $A \in R$, where $R = conv\{H^n, n = 1, 2, \ldots\}$. For any $H \in \Gamma$ there exists $B \in \mathfrak{B}$ s.t.

$Bx = BHx$ for any $x \in l_\infty$. Denote by $\mathfrak{B}(H)$ the set of all Banach limits satisfying this condition. The Cesaro operator C and the dilations operators σ_n belong to Γ.

We study the sets $\mathfrak{B}(C)$ and $\mathfrak{B}(\sigma_n)$, $n \in \mathbb{N}$ and the extremal points of $\mathfrak{B}$. We present some applications of Banach limits to the operator theory.

Joint work with E. A. Alekhno, F. A. Sukochev, A. S. Usachev. This work is supported by RNF, grant 16–11–101–25.

Order isomorphisms of operator intervals

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A general theory of order isomorphisms of operator intervals will be presented. In unifies and extends several known results, among others the famous Ludwig's description of ortho-order automorphisms of effect algebras and Molnár's characterization of bijective order preserving maps on bounded observables. The optimality of the presented theorems can be demonstrated by using Löwner's theory of operator monotone functions.

The Regular Algebra Numerical Range

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Let E be a Dedekind complete Banach lattice. For a regular operator, $T \in \mathcal{L}_r(E)$, we have the usual spatial numerical range $V(T)$ and algebra numerical range $V(\mathcal{L}(E), T)$. In this talk we will present results about the regular algebra numerical range $V(\mathcal{L}_r(E), T)$. In particular we will discuss the case $T \in \mathcal{Z}(E)$, where $\mathcal{Z}(E)$ denotes the center of E, and the case $T \perp I$.

Unbounded topologies and uo-convergence in locally solid vector lattices

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As generalizations of convergence in measure and convergence almost everywhere, respectively, unbounded norm convergence and unbounded order convergence have attracted the study of many researchers. In this talk we discuss unbounded τ-convergence where τ is a locally solid topology and show that, qualitatively, the theory remains the same independent of the choice of τ. In other words, we show that many of the results for unbounded norm convergence still hold true if the norm topology is replaced by an arbitrary locally solid topology. We also discuss the connection between unbounded order convergence, unbounded order continuous topologies, and minimal topologies.

The almost-invariant subspace problem for Banach spaces

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We show that any bounded operator acting on an infinite dimensional Banach space admits a rank one perturbation that has in invariant subspace of infinite dimension and codimension. This extends to arbitrary Banach spaces a previous result

that was proved only in the reflexive case.

Weak compactness in Banach lattices

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We will present some recent progress in weak compactness in Banach lattices. Part of the talk will be devoted to understand the minimal size of a weakly compact subset in $C[0,1]$ which cannot be covered by a reflexive Banach lattice. This is related to the factorization of weakly compact operators between Banach lattices. We will also present some results about weakly compact generation in Banach lattices and a question of J. Diestel.

Positive linear maps on C^*-algebras -Choi's conjecture-

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Let a linear map Φ between two unital C^*-algebras be positive and preserve the identity. Kadison showed that if $f(t) = |t|$ and $\Phi(f(A)) = f(\Phi(A))$ for all selfadjoint operators $A \in \mathcal{A}$, then $\Phi(A^2) = \Phi(A)^2$ for all selfadjoint operators A, that is, Φ is a C^*-homomorphism. Choi proved this fact for an operator convex function f, and then conjectured that this fact would hold for a non-affine continuous function f. We shall prove a refinement of his conjecture.

Some approaches to inverses of disjointness preserving operators on pre-Riesz spaces

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One of the astonishing results in the theory of disjointness preserving operators is that the inverse of a disjointness preserving bijection on a Banach lattice is disjointness preserving. It is tempting to try to generalize this result to a setting with more general partially ordered vector spaces. That turns out to be a difficult task. We will discuss several approaches and some results for very special cases. Also, we will illustrate some of the difficulties by examples.

This is joint work with Anke Kalauch (TU Dresden) and Feng Zhang (Leiden University).

Closed Graph Theorems for Operators on Ordered Vector Spaces

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The Closed Graph Theorem is one of the cornerstones of linear functional analysis in Fréchet spaces, and the extension of this result to more general topological vector spaces is a nontrivial problem involving significant technical difficulty. However, the theory of convergence vector spaces provides a natural framework for Closed Graph Theorems. In this talk we use techniques from convergence vector space theory to investigate versions of the Closed Graph Theorem for operators on (real) ordered vector spaces.

Automatic Linearity of Order Isomorphisms

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The famous Mazur-Ulam theorem states that any surjective isometry between two normed space is affine linear. One can wonder about the order theoretic analogy of this theorem. More precisely, letting (X, C) and (Y, K) be partially ordered vector space and $f : X \to Y$ an order isomorphism is then f necessarily affline linear?
Does this not hold generally. Howoever, in 1977 Walter Noll and Juan Schaäffer found a class of partially ordered vector spaces for which the statement does hold, namely the cone of the space needs to contain sufficiently many extreme rays. Their theory will be expanded upon in this talk by considering a more general class of partially ordered vector spaces and discussing a special shape that order isomorphisms have on such spaces.

Amenability of locally compact quantum groups and their unitary co-representations

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We will present a characterization of amenability of a locally compact quantum group by nuclearity of the its "reduced group C^*-algebra" and the existence of a state of this algebra that satisfies a trace-like property. A related result on amenability of unitary (co-) representations will be discussed.

All relevant notions will be explained during the talk.

Mixing inequalities in Riesz spaces

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Various topics in stochastic processes have been considered in the abstract setting of Riesz spaces, for example martingales, martingale convergence, ergodic theory, AMARTS, Markov processes and mixingales. Here we continue the relaxation of conditional independence begun in the study of mixingales and study mixing processes. The two mixing coefficients which will be considered are the α (strong) and φ (uniform) mixing coefficients. We conclude with mixing inequalities for these types of processes. In order to facilitate this development, the study of generalized L^1 and L^∞ spaces begun by Kuo, Labuschagne and Watson will be extended.

This is joint work with: Wen-Chi Kuo & Michael Rogans

The Riesz-Kantorovich formulae

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From the earliest days of study of ordered vector spaces in the 1930's, it has been known that if Y is a Dedekind complete Riesz space then the regular operators from any Riesz space X into Y is a lattice and that the lattice operations there are given by the Riesz-Kantorovich formulae. Since then, several other situations have been found where the regular operators form a lattice. In all of these cases, the lattice operations are given by the Riesz-Kantorovich formulae. In this talk I will survey our current knowledge concerning the Riesz-Kantorovich formulae for operators between Banach lattices. The talk will be in two parts. The first deals with situations where

the regular operators form a lattice whilst the second will address the question of whether the modulus of a single operator, if it exists, is necessarily given by the Riesz-Kantorovich formula.

Very few of the results I will present are my own, but several are due to a rather eccentric student of mine, Michael Elliott, who is unlikely to be known to many of you.

Weak containment by restrictions of induced representionss

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QSIN groups are a class of locally compact groups which include every discrete group and every amenable group. The presenter has shown that if H is a closed subgroup of a QSIN group G, then the restriction of the induced representation $\mathrm{Ind}_H^G \pi$ to H weakly contains π for every unitary representation π of H. This talk will explain this result and its applications to the local properties of group C*-algebras of QSIN groups. These local properties (local reflexivity and the local lifting property) are important C*-algebraic properties which are defined in terms of completely positive maps.

Appropriate definitions will be given during the talk.

Density of cones and the projective tensor product

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This talk consists of two parts. In the first part, we consider the question when a cone is dense in the whole space in some linear Hausdorff topology. We show the relation between this question and the existence of positive linear functions and we give some examples, using a generalization of the lexicographic cone. In the second part, we look at the projective tensor product of ordered vector spaces. In the Archimedean case it is easy to see that the cone of this projective tensor product is actually a pointed cone, but this is not so clear in the nonarchimedean case. We will give a simple proof of this fact using the above mentioned lexigraphic cones.

The uo-dual of a Banach lattice

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Let X be a Banach lattice and X' the algebraic dual of X. Then the uo-dual of X is defined as follows

$$X_{uo}^{\sim} = \{\phi \in X' \mid \phi(x_a) \to 0 \text{ whenever } x_a \xrightarrow{uo} 0 \text{ and } (x_a) \text{ is norm bounded}\}$$

In this talk we will discuss the main properties of the uo-dual and applications to the Fenchel-Moreau duality theory of convex functionals.

This talk is based on joint work with Niushan Gao and Denny H. Leung

In Search of Suitable Spaces of Continuous Functions

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Our goal in this talk is to discuss an extension of a theorem known as the Portemanteau theorem (see Klenke, Theorem 13.16, pp. 254-255 of [2]).

Let (X, d) be a Polish space, let $\mathcal{B}$ be the Borel σ-algebra on X, let $\mathcal{M}(X)$ be the usual Banach space of all real valued signed Borel measures on X, and let $C_b(X)$ be the customary Banach space of all real valued continuous bounded functions defined on X. Unexplained terminology used here can be found in [4] and [3], and will be defined in the talk.

The Portemanteau theorem consists of several equivalent characterizations of the $C_b(X)$-weak convergence of a sequence of probability measures in $\mathcal{M}(X)$.

In our approach, we consider a new type of convergence of sequences of probability measures in $\mathcal{M}(X)$, and we obtain several characterizations of this convergence.

The convergence that we define involves two spaces of continuous functions:

$C_{\rm bs}^{\rm (b)}(X)$ = the vector space of all continuous bounded real valued functions on X, functions that have bounded supports

and

$C_{\rm bs}^{\rm (ucb)}(X)$ = the vector subspace of $C_{\rm bs}^{\rm (b)}(X)$ formed of all uniformly continuous functions.

We will conclude the talk with open questions that explain the motivation for obtaining our extension of the Portemanteau theorem.

The questions are related to the ergodic decomposition defined by a transition probability on a Polish space.

The title of the talk stems from the fact that, for the ergodic decomposition, we need a vector space of continuous functions that could be used to replace $C_K(X)$ (= the vector space of all continuous real valued functions with compact support defined on X) that is employed when X is a locally compact separable metric space.

References

[1] P. Billingsley, *Convergence of Probability Measures,* Second Edition, Wiley, New York and Toronto, 1999.

[2] A. Klenke, *Probability Theory: A Comprehensive Course,* Second Edition, Springer, London, 2014.

[3] T. Szarek and R. Zaharopol, *Feller Transition Functions, Generators, and the Ergodic Decomposition,* submitted.

[4] R. Zaharopol, *Invariant Probabilities of Transition Functions,* Springer International Publishing, Switzerland, 2014.

On Dunford-Pettis-like functions on Banach lattices

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In this talk, we introduce the so-called Dunford-Pettis-type functions. We briefly explore elementary properties of polynomials and holomorphic functions found in the literature. We then introduce the so-called disjoint Dunford-Pettis property of order p and the disjoint Dunford-Pettis* property of order p, and characterise these two properties in terms of sequences. The disjoint Dunford-Pettis property of order p is then also characterised in terms of disjoint p-convergent operators. Following the definition of a p-convergent function, we introduce the so-called disjoint p-convergent functions.

We then provide conditions under which a polynomial, holomorphic function and a symmetric separately compact bilinear map are disjoint p-convergent.

Disjointness preserving C_0-semigroups and local operators on ordered Banach spaces

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We generalize results concerning C_0-semigroups on Banach lattices to a setting of ordered Banach spaces. We prove that the generator of a disjointness preserving C_0-semigroup is local. Some basic properties of local operators are also given. We investigate cases where local operators generate local C_0-semigroups, by using Taylor series or Yosida approximations. As norms we consider regular norms and show that bands are closed with respect to such norms. Our proofs rely on the theory of embedding pre-Riesz spaces in vector lattices and on corresponding extensions of regular norms.

Lie-Trotter product formula for locally equicontinuous Markov operators

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In this talk I will present a Lie-Trotter product formula for Markov semigroups on spaces of measures. We relate our results to those of strongly continuous semigroups and show that our approach is an extension of exsisting results. As Markov semigroups are usually neither strongly continuous nor bounded, we prove the convergence of Lie-Trotter product formula under weaker assumptions. We replace the strong continuity of semigroups by local equicontinuity and tightness. We also remove all the "classical" assumptions on generators and domains of generators of semigroups. A crutial tool we use in the proofs is a Schur-like property for spaces of measures.

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