University of Alberta
Mathematical and Statistical Sciences
Dr. Thomas Hillen
492-3395, thillen@ualberta.ca April 6, 2004

Math 337, Partial Differential Equations
Winter 2004

Review

The final exam will cover the material of the whole course. As a reminder I summarize the keywords from the material before the midterm. This material was reviewed in an earlier review session on February 23, 2004

1. **Before Midterm:** Classification of PDE, elliptic, parabolic, hyperbolic; polar, cylindrical, spherical coordinates; steady states, equilibrium solutions; conservation of energy; method of characteristics; D'Alembert solution; some linear algebra; PWS-functions; Fourier series; Fourier sine series; Fourier cosine series; linear operators; separation of variables for the heat equation and for the wave equation with all kind of boundary conditions; in 1-D and in 2-D; Laplace equation

2. **Calculus of Fourier series:** Differentiation term-by-term; Integration.

3. **Separation, nonhomogeneous equations:**
Review example 1: Solve

$$\begin{aligned} \frac{\partial^2 u}{\partial t^2} &= 25\frac{\partial^2 u}{\partial x^2} + t^2 \sin(x), \qquad 0 \le x \le 2\pi \\ u(0,t) &= 0 = u(2\pi,t) \\ u(x,0) &= 3\sin(2x) \\ \frac{\partial u(x,0)}{\partial t} &= 7\sin(2x) \end{aligned}$$

4. **Separation; nonhomogeneous boundary conditions:**
Review example 2: Solve

$$\begin{aligned} \frac{\partial u}{\partial t} &= c\frac{\partial^2 u}{\partial x^2}, \qquad 0 \le x \le 1 \\ \frac{\partial u(0,t)}{\partial x} &= d = \frac{\partial u(2\pi,t)}{\partial x} \\ u(x,0) &= \cos(\pi x) \end{aligned}$$

5. **both: nonhomogeneous equation and nonhomogeneous boundary conditions**

6. **Sturm-Liouville eigenvalues problems** Definition; Spectral Theorem;
Review example 3: The equation for heat flow in a non-uniform rod of length 2 with leaking ends is described by

$$c(x)\rho(x)\frac{\partial u}{\partial t} = \frac{\partial}{\partial x}\left(K_0(x)\frac{\partial u}{\partial x}\right) + \alpha u$$

$$u(0,t) = -\frac{\partial}{\partial x}u(0,t) \qquad u(2,t) = \frac{\partial}{\partial x}u(2,t),$$

with physical parameter functions $c(x), \rho(x), K_0(x) > 0$, and $\alpha > 0$. Use separation and show that the spatial problem is a Sturm-Liouville problem (Note: you do not need to solve this equation!)

7. **Rayleigh quotient:** You need to be able to write it down!
   Review example 4: Estimate the leading eigenvalue of

$$\begin{aligned} \varphi'' - x\varphi + \lambda\varphi &= 0 \\ \varphi'(0) + \varphi(0) = 0 \qquad & \varphi'(1) = 0. \end{aligned}$$

8. **General PDE example, generalized Fourier-series**

9. **Classical PDE problems**, Laplace equation in polar coordinates; Fisher equation

10. **Fourier transform** Fourier integral formula, complex formulations; Fourier-sine and -cosine integral formulas; Fourier transform.
    Review example 5: Find (a) the Fourier transform, (b) the Fourier-sine integral and (c) the Fourier cosine integral of

$$f(x) = \begin{cases} 0 & x < 0 \\ 1 & 0 < x < 1 \\ 2 & 1 < x < 2 \\ 0 & x > 2 \end{cases}$$

11. **Fourier transform methods for PDE:** Gauss kernel, heat kernel, convolution formula for the heat equation, D'Alemberts formula for the wave equation.

    Review example 6: Use the error-function to solve

$$\begin{aligned} \frac{\partial u}{\partial t} &= \frac{\partial^2 u}{\partial x^2}, \qquad \infty < x < \infty \\ u(x,0) &= \begin{cases} 100, & |x| < 1 \\ 0 & |x| > 1 \end{cases} \end{aligned}$$

12. **Laplace transform methods for PDE:**
    Review example 7:

$$\begin{aligned} \frac{\partial^2 u}{\partial t^2} &= \frac{\partial^2 u}{\partial x^2}, \qquad 0 \le x < \infty \\ u(0,t) &= 3\cosh(2t) \\ \lim_{x\to\infty} u(x,t) &= 0 \\ u(x,0) &= 3e^{-2x} \\ \frac{\partial u(x,0)}{\partial t} &= 0 \end{aligned}$$