Math 209: Old Final Exam Questions

1. (a) Find the mass and the moment of inertia about the y-axis for a plate with constant density k which occupies the region under the curve $y = \sin x$, above the x-axis from $x = 0$ to $x = \pi$.
   Note: $\int x^2 \sin x dx = -x^2 \cos x + 2x \sin x + 2 \cos x$.

   (b) Evaluate
   $$\int_0^8 \int_{y^{1/3}}^2 e^{x^4} dxdy.$$

2. Find $\int\int\int_R zdV$, where R is the region satisfying
   $$\sqrt{3(x^2+y^2} \leq z \leq \sqrt{2-x^2-y^2}.$$

3. Let $\vec{F} = yz \cos x\vec{i} + z \sin x\vec{j} + (y \sin x + 2z)\vec{k}$ and let C be the curve
   $$\vec{r}(t) = \langle \frac{\pi}{2}t^2, e^{t^2}, \cos^4(t\pi) \rangle, \quad 0 \leq t \leq 1.$$
   Evaluate $\int_C \vec{F} \cdot d\vec{r}$.

4. Evaluate $\int\int_S (x^2+y^2)dS$ where S is the surface consisting of the part of the cone $z^2 = 3(x^2+y^2)$ that lies above $z = x^2+y^2$.

5. (a) Evaluate the line integral
   $$\int_C (y + e^{\sqrt{x}})dx + (2x + \cos(y^2))dy$$
   where C is the curve bounding the region enclosed by $y = x^2$ and $x = y^2$, traversed counterclockwise.

   (b) Find the area of a region bounded by the curve C with parametric equation
   $$x = \cos(t), y = \sin^3(t), 0 \leq t \leq 2\pi.$$

6. Let D be the solid enclosed by the surfaces: $2y = \sqrt{x^2+z^2}, y = 1, y = 2$ and let S be the boundary of D. Find the outward flux:
   $$\int\int_S \vec{F} \cdot \vec{n} dS$$
   if $\vec{F} = (x^2 + y\sin(z))\vec{i} - (y^2 + z\sin(x))\vec{j} + (z^2 + x\sin(y))\vec{k}$ and $\vec{n}$ is the outward unit normal to D.

7. (a) Verify that the function $u(x,t) = \sin(x - at)$ satisfies the wave equation

$$u_{tt} = a^2 u_{xx}.$$

(b) Find the equation of the tangent plane at the point $(-2, 2, -3)$ to the ellipsoid

$$\frac{x^2}{4} + y^2 + \frac{z^2}{9} = 3.$$

8. (a) Find the volume of a tetrahedron bounded by the planes $x + 2y + z = 2, x = 2y, x = 0$ and $z = 0$.

(b) Use polar coordinates to combine the sum and evaluate it

$$\int_{1/\sqrt{2}}^{1} \int_{\sqrt{1-x^2}}^{x} xydydx + \int_{1}^{\sqrt{2}} \int_{0}^{x} xydydx + \int_{\sqrt{2}}^{2} \int_{0}^{\sqrt{4-x^2}} xydydx.$$

9. (a) Evaluate $\int_c y \sin z ds$, where C is the circular helix given by the equations

$$x = \cos t, y = \sin t, z = t, 0 \le t \le 2\pi.$$

(b) Find the flux of the vector field $\vec{F}(x,y,z) = z\vec{i} + y\vec{j} + x\vec{k}$ over the unit sphere $x^2 + y^2 + z^2 = 1$.

10. Calculate the work done by a force field

$$\vec{F} = (x^x + z^2)\vec{i} + (y^y + x^2)\vec{j} + (z^z + y^2)\vec{k}$$

when a particle moves under its influence around the edge of the part of the sphere $x^2 + y^2 + z^2 = 4$ that lies in the first octant.

11. Evaluate

$$\int_C (y + \sin x)dx + (x^2 + \cos y)dy + x^3 dz$$

where C is the curve

$$\vec{r}(t) = \sin t\vec{i} + \cos t\vec{j} - \cos(2t)\vec{k}$$

with $0 \le t \le 2\pi$. (Hint: Use Stokes theorem given that C lies on the surface $z = x^2 - y^2$.)