Math 209

Assignment 6

Due: 12 Noon on Thursday, November 3, 2005

1. Evaluate the following triple integral:

$$\int_0^a \int_0^x \int_0^{xy} x^3 y^2 z \, dz dy dx.$$

2. Evaluate the following integral

$$\iiint_\Omega \frac{1}{(x+y+z+1)^3} dV,$$

where Ω is the tetrahedron bounded by the planes $x = 0$, $y = 0$, $z = 0$ and $x + y + z = 1$.

3. Consider the following iterated integral:

$$\int_0^1 \int_0^{1-x^2} \int_0^{1-x^2-y} f(x,y,z) dz \, dy \, dx. \tag{1}$$

   (a) Identify the region D of integration (show this region on a sketch).

   (b) Rewrite the integral (1) as an equivalent iterated integral in the five other orders.

4. Find the moment M_{xy} of the solid E described by the inequalities

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} \le 1, \quad z \ge 0,$$

i.e. E is the upper half of the solid ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} \le 1$.

5. Compute the following integral (by changing to cylindrical or spherical coordinates)

$$\int_0^2 \int_0^{\sqrt{2x-x^2}} \int_0^a z\sqrt{x^2+y^2} \, dz \, dy \, dx. \tag{2}$$

6. Compute the following integral (by changing to cylindrical or spherical coordinates)

$$\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \sqrt{x^2+y^2+z^2} \, dz \, dy \, dx. \tag{3}$$

7. Use the cylindrical coordinates to (i) identify the solid bounded by the surface

$$(x^2+y^2+z^2)^3 = a^2(x^2+y^2)^2, \tag{4}$$

and (ii) compute the volume of this solid.

8. Evaluate $\iiint_E (x^3 + xy^2) dV$, where E is the solid in the first octant that lies beneath the paraboloid $z = 1 - x^2 - y^2$.

9. Find the volume of the smaller wedge cut from a sphere of radius a by two planes that intersect along a diameter at an angle of $\frac{\pi}{6}$.

10. Evaluate the integral

$$\int_0^3 \int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} (x^3 + xy^2) dy \, dx.$$