

UNIVERSITY OF ALBERTA  
MATHEMATICAL AND STATISTICAL SCIENCES  
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**Math 300, Advanced Boundary Value Problems**  
Fall 2006

**Midterm**

Wednesday Oct 25, 2006, 1:00-1:50 PM.

<b>NAME:</b>		<b>ID:</b>	
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Exercise	max	points
1	25	
2	25	
3	50	
Total	100	

**Problem 1****[25]**

(a) Show that the Chebychev equation

$$(1 - x^2)u'' - xu' + \lambda u = 0, \quad 0 \leq x \leq 1,$$

is not of Sturm-Liouville form.

(b) Multiply the Chebychev equation by  $\frac{1}{\sqrt{1-x^2}}$  and show that the resulting equation is of Sturm-Liouville type. Identify  $p, q, \sigma$ .

**Problem 2****[25]**

Give lower and upper bounds for the leading eigenvalue of the Legendre equation

$$\begin{aligned}(1 - x^2)u'' - 2xu' + \lambda u &= 0 & 0 \leq x \leq 1, \\ u(0) = 0, \quad u'(1) &= -u(1).\end{aligned}$$

**Problem 3****[50]**

Solve the Laplace equation inside the semicircle of radius  $0 \leq r \leq 1$  and angle  $0 \leq \theta \leq \pi$ , where the diameter is perfectly insulated, and  $u(1, \theta) = 2 + \cos 2\theta$ .

(Hint: A Cauchy-Euler equation can be solved by functions of the form  $G(r) = r^\alpha$ .)