University of Alberta
Mathematical and Statistical Sciences
Dr. T. Hillen November 29, 2006

Final Exam, Math 300, Winter 05

Problem 1: Find the solution $u(r,\theta)$ to the Laplace equation in ploar coordinates given by

$$u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\theta\theta} = 0, \quad 1 < r < 2, \quad 0 < \theta < \frac{\pi}{2}$$

$$u(1,\theta) = 15\sin(2\theta), \quad u(2,\theta) = 255\sin(4\theta), \quad 0 < \theta < \frac{\pi}{2}$$

$$u(r,0) = u(r,\frac{\pi}{2}) = 0, \quad 1 < r < 2.$$

Problem 2: Show that

$$\int_0^\infty \frac{\cos(\theta x)}{a^2+\theta^2}d\theta = \frac{\pi\exp(-a|x|)}{2a}, \quad a > 0, \quad -\infty < x < \infty.$$

Problem 3: Find the solution to the wave equation given by

$$u_{tt} = u_{xx}, \quad 0 < x < 1, \quad t > 0,$$

$$u(0,t) = 0, \quad u(1,t) = \frac{sin(2\pi t)}{2\pi}, \quad u(x,0) = u_t(x,0) = 0.$$

Problem 4: Show, by determining the *Fourier Transform Solution*, that

$$u_{tt} - c^2u_{xx} = 0, \quad -\infty < x < \infty, \quad t > 0,$$

$$u(x,0) = f(x), \quad u_t(x,0) = 0, \text{ where } \int_{-\infty}^{\infty} |f(x)|dx < \infty.$$

is solved by

$$u(x,t) = \frac{f(x-ct)+f(x+ct)}{2}$$

Note: In last years course we used a different definition of the Fourier transform !!! Which means the solutions need to be adapted to use **our definition of:**

$$\mathcal{F}(f) = \frac{1}{2\pi}\int_{-\infty}^{\infty} f(x)e^{i\omega x}dx$$

and

$$\mathcal{F}^{-1}(g) = \int_{-\infty}^{\infty} g(\omega)e^{-i\omega x}dx$$

I will present the solutions in class using our definitions.