(1)

2.5.1 Laplace equation $0 \leq x \leq L$ $0 \leq y \leq H$

(a) $u(0,y) = g(y)$ $u(L,y) = 0$ $\frac{\partial u}{\partial y}(x,0)$ $u(x,H) = 0$

$\nabla^2 u = 0$ $u(x,y) = h(x)\phi(y)$ $\Rightarrow$ $\begin{cases} \phi'(0) = 0 \\ \phi(H) = 0 \end{cases}$

$\nabla^2 u = 0 \Rightarrow h''(x)\phi(y) + \phi''(y)h(x) = 0$

$\Rightarrow \frac{h''(x)}{h(x)} = \frac{-\phi''(y)}{\phi(y)} = \lambda \Rightarrow \begin{cases} h''(x) = \lambda h(x) \\ \phi''(y) = -\lambda\phi(y) \end{cases}$

$\boxed{\begin{array}{c} \phi''(y) = -\lambda\phi(y) \\ \phi'(0) = 0 = \phi(H) \end{array}}$ and $\boxed{\begin{array}{c} h''(x) = \lambda h(x) \\ h(L) = 0 \end{array}}$

$\rightarrow r^2 = -\lambda$

If $\lambda < 0 \Rightarrow \phi(y) = c_1 e^{y\sqrt{-\lambda}} + c_2 e^{-y\sqrt{-\lambda}}$

$\phi'(y) = \left(c_1 e^{y\sqrt{-\lambda}} - c_2 e^{-y\sqrt{-\lambda}}\right)\sqrt{-\lambda}$

$\phi'(0) = 0 \Rightarrow c_1 = c_2$

$\phi(H) = 0 \Rightarrow c_1 = 0 \Rightarrow$ trivial solution

If $\lambda = 0 \Rightarrow \phi(y) = ay + b$

$\left.\begin{array}{l} \phi(H) = 0 \Rightarrow aH + b = 0 \\ \phi'(0) = a = 0 \Rightarrow a = 0 \end{array}\right\} \Rightarrow b = 0$ trivial

$\lambda > 0$ $\phi(y) = a\sin(y\sqrt{\lambda}) + b\cos(y\sqrt{\lambda})$

$\phi'(y) = a\sqrt{\lambda}\cos(y\sqrt{\lambda}) - b\sqrt{\lambda}\sin(y\sqrt{\lambda})$

$\phi'(0) = 0 \Rightarrow a = 0$

$\phi(H) = 0 \Rightarrow b\cos(H\sqrt{\lambda}) = 0 \Rightarrow H\sqrt{\lambda} = \frac{(2n-1)\pi}{2}$ $n = 1, 2, 3$

2

$$\Rightarrow \boxed{\lambda_n = \left(\frac{(2n-1)\pi}{2H}\right)^2}, \quad \boxed{\phi_n(y) = b_n \cos\left(\frac{(2n-1)\pi}{2H} y\right)}$$

$$h''(x) = \lambda_n h(x) \qquad h''(x) = \left(\frac{2n-1}{2H}\pi\right)^2 h(x)$$

$$h(L) = 0$$

$$\Rightarrow h(x) = a_1 \cosh \frac{(2n-1)\pi}{2H}(x-L) + a_2 \sinh \frac{(2n-1)\pi}{2H}(x-L)$$

$$h(L) = 0 \quad \Longrightarrow \quad a_1 \cosh\left(\frac{(2n-1)\pi(0)}{2H}\right) + a_2 \sinh\left(\frac{(2n-1)\pi}{2H}(0)\right) = 0$$

$$\Longrightarrow a_1 = 0$$

$$\boxed{h_n(x) = a_n \sinh\left(\frac{(2n-1)\pi}{2H}(x-L)\right)}$$

$$\boxed{u(x,y) = \sum_{n=1}^{\infty} A_n \cos\left(\frac{(2n-1)}{2H}\pi y\right) \sinh\left[\left(\frac{2n-1}{2H}\right)\pi(x-L)\right]}$$

$$u(0,y) = g(y)$$

$$u(0,y) = \sum_{n=1}^{\infty} A_n \cos\left(\frac{2n-1}{2H}\pi y\right) \sinh\left(\frac{-(2n-1)\pi L}{2H}\right) = g(y)$$

$$\int_0^H g(y) \cos\left(\frac{2n-1}{2H}\pi y\right) dy = \int_0^H \sum_{n=1}^{\infty} A_n \cos\left(\frac{2n-1}{2H}\pi y\right) \cos\left(\frac{(2m-1)\pi y}{2H}\right) \sinh\left(\frac{(1-2n)\pi L}{2H}\right)$$

using the orthogonality of cosine we have:

$$\boxed{A_m = \frac{2}{L \sinh\left(\frac{(1-2m)\pi L}{2H}\right)} \int_0^H g(y) \cos\left(\frac{(2m-1)\pi y}{2H}\right) dy}$$

③

2.5.1 (f) $u(0,y) = f(y)$ $u(L,y) = 0$ $\frac{\partial u}{\partial y}(x,0) = 0 = \frac{\partial u}{\partial y}(x,H)$

$u(x,y) = \phi(y)h(x)$, $\nabla^2 u = 0 \Rightarrow \frac{h''(x)}{h(x)} = \frac{-\phi''(y)}{\phi(y)} = \lambda$

$\phi'(H) = 0 = \phi'(0)$ $h(L) = 0$

$\phi''(y) = -\lambda\phi(y)$

$\phi'(H) = 0$

$\phi'(0) = 0$

$\underline{\lambda < 0}$ $\phi(y) = c_1 e^{y\sqrt{\lambda}} + c_2 e^{-y\sqrt{\lambda}}$ apply $\phi'(0) = \phi'(H) = 0 \Rightarrow$ trivial case

$\underline{\lambda = 0}$ $\phi(y) = ay + b$ $\phi'(y) = a$

$\left.\begin{array}{l}\phi'(0) = 0 \\ \phi'(H) = 0\end{array}\right\} \Rightarrow a = 0$ $\phi(y) =$ Constant is nontrivial

$\underline{\lambda > 0}$ $\phi(y) = a\sin(y\sqrt{\lambda}) + b\cos(y\sqrt{\lambda})$

$\phi'(y) = a\sqrt{\lambda}\cos(y\sqrt{\lambda}) - b\sqrt{\lambda}\sin(y\sqrt{\lambda})$

$\phi'(0) = 0 \Rightarrow a = 0$

$\phi'(H) = 0 \Rightarrow -b\sqrt{\lambda}\sin(H\sqrt{\lambda}) = 0 \Rightarrow \lambda = \left(\frac{n\pi}{H}\right)^2$

$$\boxed{\phi_n(y) = b_n \cos\left(\frac{n\pi y}{H}\right)}$$

$\begin{array}{l} h''(x) = \lambda h(x) \\ h(L) = 0 \end{array} \Rightarrow \boxed{h_n(x) = a_n \sinh\left(\frac{n\pi(x-L)}{H}\right)}$

$$\boxed{u(x,y) = A_0 + \sum_{n=1}^{\infty} A_n \cos\left(\frac{n\pi y}{H}\right)\sinh\left(\frac{n\pi(x-L)}{H}\right)}$$

$$u(0,y) = f(y) = A_0 + \sum_{n=1}^{\infty} A_n \cos\left(\frac{n\pi y}{H}\right)\sinh\left(\frac{-n\pi L}{H}\right)$$

④

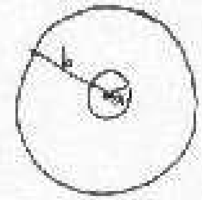
using the orthogonality of Cosines:

$$\boxed{A_n = \frac{2}{L\sinh\left(\frac{-n\pi L}{H}\right)}\int_0^H f(y)\cos\left(\frac{n\pi y}{H}\right)dy}$$

$$\boxed{A_0 = \frac{1}{L\sinh\left(\frac{-n\pi L}{H}\right)}\int_0^H f(y)\,dy}$$

$$\boxed{u(x,y) = A_0 + \sum_{n=1}^{\infty} A_n \cos\left(\frac{n\pi y}{H}\right)\sinh\left(\frac{n\pi(x-L)}{H}\right)}$$

2.5.8 (b) $\quad a<r<b$



$$\frac{\partial u}{\partial r}(a,\theta)=0 \qquad u(b,\theta)=g(\theta)$$

$$\boxed{\nabla^2 u=\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2}=0} \quad \text{(I)} \qquad -\pi<\theta<\pi$$

$$u(r,-\pi)=u(r,\pi)$$

$$\frac{\partial u}{\partial \theta}(r,-\pi)=\frac{\partial u}{\partial \theta}(r,\pi) \qquad \frac{\partial u}{\partial r}(a,\theta)=0 \Rightarrow G'(a)=0$$

$$u(r,\theta)=\phi(\theta)G(r) \qquad \phi(-\pi)=\phi(\pi)$$

$$\phi'(-\pi)=\phi'(\pi)$$

plug into (I)

$$\frac{r}{G}\frac{d}{dr}\left(r\frac{dG}{dr}\right)=\frac{-1}{\phi}\frac{d^2\phi}{d\theta^2}=\lambda$$

$$\frac{d^2\phi}{d\theta^2}=-\lambda\phi$$

$$\phi(-\pi)=\phi(\pi) \qquad \Longrightarrow \qquad \lambda_n=\left(\frac{n\pi}{\pi}\right)^2=n^2$$

$$\frac{d\phi}{d\theta}(-\pi)=\frac{d\phi}{d\theta}(\pi)$$

$$\phi_n(\theta)=a_1\sin n\theta+a_2\cos n\theta$$

$$\frac{r}{G}\frac{d}{dr}\left(r\frac{dG}{dr}\right)-n^2G=0$$

for $n\neq 0 \qquad G_1=c_1r^n+c_2r^{-n}$

for $n=0 \qquad G_2=\bar{c}_1+\bar{c}_2\ln r$

$$G_2'(a)=\bar{c}_2\frac{1}{a}=0 \Rightarrow \bar{c}_2=0 \Rightarrow G_2=\bar{c}_1$$

$$G_1'(r)=nc_1r^{n-1}-nc_2r^{-n-1} \Rightarrow G_1'(a)=0 \Rightarrow nc_1a^{n-1}-nc_2a^{-n-1}=0$$

$$\Rightarrow \quad n c_1 a^{n-1} - \frac{n c_2}{a^{n+1}} = 0 \quad \Rightarrow \quad \frac{n c_1 a^{2n} - n c_2}{a^{n+1}} = 0$$

$$\Rightarrow \quad n(c_1 a^{2n} - c_2) = 0 \quad \Rightarrow \quad c_1 a^{2n} = c_2$$

$$\boxed{G(r) = C\left(r^n + a^{2n} r^{-n}\right)}$$

$$\boxed{u(r,\theta) = \sum_{n=0}^{\infty} A_n \left(r^n + a^{2n} r^{-n}\right) \cos n\theta + \sum_{n=1}^{\infty} B_n \left(r^n + a^{2n} r^{-n}\right) \sin n\theta}$$

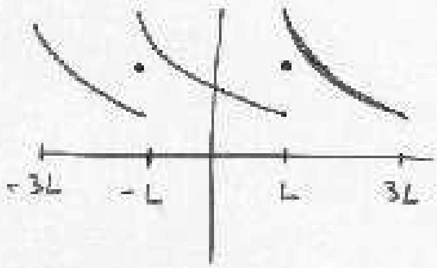
$$g(\theta) = u(b,\theta) = \sum_{n=0}^{\infty} A_n \left(b^n + a^{2n} b^{-n}\right) \cos n\theta + \sum_{n=1}^{\infty} B_n \left(b^n + a^{2n} b^{-n}\right) \sin n\theta$$

$$\boxed{\begin{aligned} A_0 &= \frac{1}{2\pi} \int_{-\pi}^{\pi} g(\theta)\, d\theta \\ A_n &= \frac{1}{\pi\left(b^n + a^{2n} b^{-n}\right)} \int_{-\pi}^{\pi} g(\theta) \cos n\theta \, d\theta \\ B_n &= \frac{1}{\pi\left(b^n + a^{2n} b^{-n}\right)} \int_{-\pi}^{\pi} g(\theta) \sin n\theta \, d\theta \end{aligned}}$$

3.2.2(b) $f(x) = e^{-x}$ $\quad -L \le x \le L$ ⑦

$$a_0 = \frac{1}{2L}\int_{-L}^{L} f(x)\,dx \qquad a_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos\frac{n\pi x}{L}\,dx$$

$$b_n = \frac{1}{L}\int_{-L}^{L} f(x)\sin\frac{n\pi x}{L}\,dx$$



$$a_0 = \frac{1}{2L}\int_{-L}^{L} e^{-x}\,dx = \frac{1}{2L}\left[-e^{-x}\right]_{-L}^{L} = \frac{1}{2L}\left[-e^{-L}+e^{+L}\right] = \frac{\sinh L}{L}$$

$$a_n = \frac{1}{L}\int_{-L}^{L} e^{-x}\cos\frac{n\pi x}{L}\,dx = \frac{(-e^{-L}L\cos(n\pi) + e^{-L}n\pi\sin n\pi + e^{L}L\cos(n\pi) + e^{L}n\pi\sin n\pi}{L^2+n^2\pi^2}$$

$$= \frac{2L(-1)^n}{L^2+n^2\pi^2}\,\frac{e^{L}-e^{-L}}{2} = \frac{2L(-1)^n}{L^2+n^2\pi^2}\sinh L$$

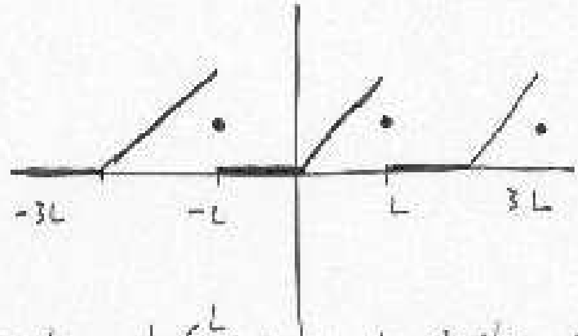
$$b_n = \frac{1}{L}\int_{-L}^{L} e^{-x}\sin\frac{n\pi x}{L}\,dx = \frac{(-e^{-L}n\pi\cos(n\pi) - e^{-L}L\sin(n\pi) + e^{L}n\pi\cos(n\pi) + e^{L}L\sin(n\pi))}{L^2+n^2\pi^2}$$

$$= \frac{2n\pi(-1)^n}{L^2+n^2\pi^2}\sinh(L)$$

(8)

3.2.2 d

$$f(x) = \begin{cases} 0 & x<0 \\ x & x>0 \end{cases}$$



$$a_0 = \frac{1}{2L}\int_{-L}^{L} f(x)\,dx = \frac{1}{2L}\int_{-L}^{0} 0\,dx + \frac{1}{2L}\int_{0}^{L} x\,dx = \frac{1}{2L}\frac{x^2}{2}\Big|_0^L = \frac{L^2}{4L} = \frac{L}{4}$$

$$a_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos\left(\frac{n\pi x}{L}\right)dx = \frac{1}{L}\int_{0}^{L} x\cos\left(\frac{n\pi x}{L}\right)dx$$

$$= \frac{L\left(\cos(n\pi) - n\pi\sin(n\pi) - 1\right)}{n^2\pi^2} = \frac{L\left((-1)^n - 1\right)}{n^2\pi^2} = \begin{cases} 0 & n \text{ even} \\ \frac{-2L}{n^2\pi^2} & n \text{ odd} \end{cases}$$

$$b_n = \frac{1}{L}\int_{-L}^{L} f(x)\sin\left(\frac{n\pi x}{L}\right)dx = \frac{1}{L}\int_{0}^{L} x\sin\left(\frac{n\pi x}{L}\right)dx$$

$$= \frac{-L\left(-\sin(n\pi) + n\pi\cos(n\pi)\right)}{n^2\pi^2} = \frac{-L\left(n\pi(-1)^n\right)}{n^2\pi^2} = \frac{Ln\pi(-1)^{n+1}}{n^2\pi^2}$$

3.2.3

let $\bar{a}_0, \bar{a}_n, \bar{b}_n$ be Fourier coefficient for $f(x)$.
let $\hat{a}_0, \hat{a}_n, \hat{b}_n$ be Fourier coefficient for $g(x)$.
let a_0, a_n, b_n be Fourier coefficient for $c_1 f(x) + c_2 g(x)$

$$a_0 = \frac{1}{2L}\int_{-L}^{L}\left(c_1 f(x) + c_2 g(x)\right)dx = \frac{1}{2L}\left(c_1\int_{-L}^{L} f(x)\,dx + c_2\int_{-L}^{L} g(x)\,dx\right)$$

$$\Rightarrow \quad \boxed{a_0 = c_1\bar{a}_0 + c_2\hat{a}_0} \quad \text{I}$$

$$a_n = \frac{1}{L}\int_{-L}^{L}\left(c_1 f(x) + c_2 g(x)\right)\cos\left(\frac{n\pi x}{2}\right)dx = \frac{c_1}{L}\int_{-L}^{L} f(x)\cos\left(\frac{n\pi x}{L}\right)dx + \frac{c_2}{L}\int_{-L}^{L} g(x)\cos\left(\frac{n\pi x}{L}\right)dx$$

$$\Rightarrow \quad \boxed{a_n = c_1\bar{a}_n + c_2\hat{a}_n} \quad \text{II}$$

Similarly: $\boxed{b_n = c_1\bar{b}_n + c_2\hat{b}_n}$ III

Fourier Series of $c_1 f + c_2 g$

$$c_1 f(x) + c_2 g(x) = \sum_{n=0}^{\infty} a_n \cos\left(\frac{n\pi x}{L}\right) + \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{L}\right) = \boxed{\text{using I II and III}}$$

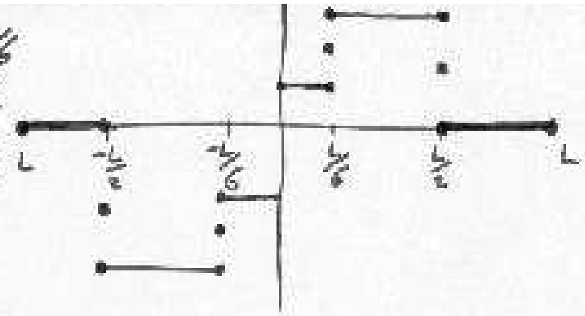
$$= \sum_{n=0}^{\infty}\left(c_1\bar{a}_n + c_2\hat{a}_n\right)\cos\left(\frac{n\pi x}{L}\right) + \sum_{n=1}^{\infty}\left(c_1\bar{b}_n + c_2\hat{b}_n\right)\sin\left(\frac{n\pi x}{L}\right)$$

$$= \sum_{n=0}^{\infty} c_1\bar{a}_n\cos\left(\frac{n\pi x}{L}\right) + \sum_{n=0}^{\infty} c_2\hat{a}_n\cos\left(\frac{n\pi x}{L}\right) + \sum_{n=1}^{\infty} c_1\bar{b}_n\sin\left(\frac{n\pi x}{L}\right) + \sum_{n=1}^{\infty} c_2\hat{b}_n\sin\left(\frac{n\pi x}{L}\right)$$

$$= c_1\left[\sum_{n=0}^{\infty}\bar{a}_n\cos\left(\frac{n\pi x}{L}\right) + \sum_{n=1}^{\infty}\bar{b}_n\sin\left(\frac{n\pi x}{L}\right)\right] + c_2\left[\sum_{n=0}^{\infty}\hat{a}_n\sin\left(\frac{n\pi x}{L}\right) + \sum_{n=1}^{\infty}\hat{b}_n\sin\left(\frac{n\pi x}{L}\right)\right]$$

$= c_1$ (Fourier series of $f(x)$) $+ c_2$ (Fourier series of $g(x)$

3.3.2(b) $f(x) = \begin{cases} 1 & x < L/6 \\ 3 & L/6 < x < L/2 \\ 0 & x > L/2 \end{cases}$



$$B_n = \frac{2}{L}\int_0^L f(x)\sin\left(\frac{n\pi x}{L}\right)dx$$

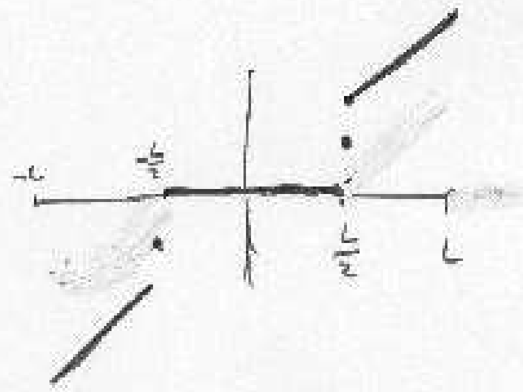
$$= \frac{2}{L}\int_0^{L/6}\sin\left(\frac{n\pi x}{L}\right) + \frac{2}{L}\int_{L/6}^{L/2} 3\sin\left(\frac{n\pi x}{L}\right)dx + \int_{L/2}^{L} 0\,dx$$

$$= \frac{2}{L}\left(-\cos\left(\frac{n\pi x}{L}\right)\cdot\frac{L}{n\pi}\right)\Bigg|_0^{L/6} + \frac{6}{L}\left(-\cos\left(\frac{n\pi x}{L}\right)\frac{L}{n\pi}\right)\Bigg|_{L/6}^{L/2}$$

$$= \frac{2}{n\pi}\left(-\cos\left(\frac{n\pi}{6}\right)+1\right) + \frac{6}{n\pi}\left[-\cos\left(\frac{n\pi}{2}\right)+\cos\left(\frac{n\pi}{6}\right)\right]$$

$$= \frac{1}{n\pi}\left[2-6\cos\left(\frac{n\pi}{2}\right)+4\cos\left(\frac{n\pi}{6}\right)\right]$$

3.3.2 (c) $f(x) = \begin{cases} 0 & x < \frac{L}{2} \\ x & x > \frac{L}{2} \end{cases}$

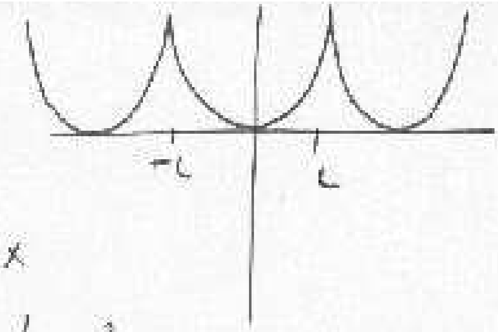


$$B_n = \frac{2}{L}\int_0^L f(x)\sin\frac{n\pi x}{L}dx = \frac{2}{L}\int_{L/2}^{L} x\sin\frac{n\pi x}{L}dx$$

$$= \frac{-L}{n^2\pi^2}\left[-2\sin(n\pi) + 2n\pi\cos(n\pi) + 2\sin\left(\frac{n\pi}{2}\right) - n\pi\cos\left(\frac{n\pi}{2}\right)\right]$$

$$= \frac{-L}{n^2\pi^2}\left[2n\pi(-1)^n + 2\sin\left(\frac{n\pi}{2}\right) - n\pi\cos\left(\frac{n\pi}{2}\right)\right]$$

3.3.5 (a) $\quad f(x) = x^2$



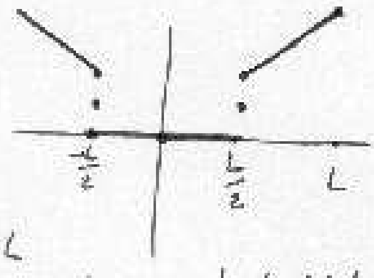
$$A_0 = \frac{1}{L}\int_0^L f(x)\,dx = \frac{1}{L}\int_0^L x^2\,dx$$

$$= \frac{1}{L}\left[\frac{x^3}{3}\right]_0^L = \frac{L^2}{3}$$

$$A_n = \frac{2}{L}\int_0^L f(x)\cos\left(\frac{n\pi x}{L}\right)dx = \frac{2}{L}\int_0^L x^2\cos\left(\frac{n\pi x}{L}\right)dx = \frac{2L^2}{n^3\pi^3}\left(n^2\pi^2\sin(n\pi) - 2\sin(n\pi) + 2n\pi\cos(n\pi)\right)$$

$$= \frac{4L^2}{n^2\pi^2}(-1)^n$$

3.3.5 (c) $\quad f(x) = \begin{cases} 0 & x < L/2 \\ x & x > L/2 \end{cases}$



$$A_0 = \frac{1}{L}\int_0^L f(x)\,dx = \frac{1}{L}\int_0^{L/2} 0\,dx + \frac{1}{L}\int_{L/2}^L x\,dx = \frac{1}{L}\left(\frac{x^2}{2}\right)\Bigg|_{L/2}^L = \frac{3L}{8}$$

$$A_n = \frac{2}{L}\int_0^L f(x)\cos\left(\frac{n\pi x}{L}\right)dx = \frac{2}{L}\int_{L/2}^L x\cos\left(\frac{n\pi x}{L}\right)dx =$$

$$\frac{L\left(2\cos(n\pi) + 2n\pi\sin(n\pi) - 2\cos\left(\frac{n\pi}{2}\right) - n\pi\sin\left(\frac{n\pi}{2}\right)\right)}{n^2\pi^2}$$

$$= \frac{L}{n^2\pi^2}\left(2(-1)^n - 2\cos\left(\frac{n\pi}{2}\right) - n\pi\sin\left(\frac{n\pi}{2}\right)\right)$$

3.3.17

$\int_0^1 \frac{dx}{1+x^2}$

(a) $\int_0^1 \frac{1}{1+x^2}\,dx = \tan^{-1}(x)\Big|_0^1 = \tan^{-1}(1) - \tan^{-1}(0) = \frac{\pi}{4}$

(b) $\frac{1}{1+x^2} = \sum_{n=0}^{\infty} x^{2n}(-1)^n = 1 - x^2 + x^4 - x^6 + \cdots$

$$\int_0^1 \frac{1}{1+x^2}\,dx = \int_0^1 \sum_{n=0}^{\infty} x^{2n}(-1)^n\,dx = \sum_{n=0}^{\infty}(-1)^n \int_0^1 x^{2n}\,dx$$

$$= \sum_{n=0}^{\infty}(-1)^n \frac{x^{2n+1}}{2n+1}\Big|_0^1 = \sum_{n=0}^{\infty}\frac{(-1)^n}{2n+1}$$

(c) $\sum_{n=0}^{\infty}(-1)^n \frac{1}{2n+1} = \frac{\pi}{4}$

$$\Rightarrow \quad \pi = 4\sum_{n=0}^{\infty}(-1)^n\frac{1}{2n+1}$$

3.4.8 (13)

let $u(x,t) = \sum_{n=0}^{\infty} A_n(t)\cos\left(\frac{n\pi x}{L}\right) + \sum_{n=1}^{\infty} B_n(t)\sin\left(\frac{n\pi x}{L}\right)$

$$\frac{\partial^2 u}{\partial x^2} = \sum_{n=0}^{\infty} -A_n(t)\cos\left(\frac{n\pi x}{L}\right)\left(\frac{n\pi}{L}\right)^2 + \sum_{n=1}^{\infty} -B_n(t)\sin\left(\frac{n\pi x}{L}\right)\left(\frac{n\pi}{L}\right)^2$$

$$\frac{\partial u}{\partial t} = \sum_{n=0}^{\infty} A_n'(t)\cos\left(\frac{n\pi x}{L}\right) + \sum_{n=1}^{\infty} B_n'(t)\sin\left(\frac{n\pi x}{L}\right)$$

$$K\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t} \Longrightarrow$$

$$K\sum_{n=0}^{\infty} -A_n(t)\cos\left(\frac{n\pi x}{L}\right)\left(\frac{n\pi}{L}\right)^2 + \sum_{n=1}^{\infty} -B_n(t)\sin\left(\frac{n\pi x}{L}\right)\left(\frac{n\pi}{L}\right)^2$$
$$= \sum_{n=0}^{\infty} A_n'(t)\cos\left(\frac{n\pi x}{L}\right) + \sum_{n=1}^{\infty} B_n'(t)\sin\left(\frac{n\pi x}{L}\right)$$

$$\Longrightarrow \begin{cases} -A_n(t)\left(\frac{n\pi}{L}\right)^2 K = A_n'(t) \Rightarrow A_n(t) = A_n e^{-\left(\frac{n\pi}{L}\right)^2 Kt} \\ -B_n(t)\left(\frac{n\pi}{L}\right)^2 K = B_n'(t) \Rightarrow B_n(t) = B_n e^{-\left(\frac{n\pi}{L}\right)^2 Kt} \end{cases}$$

$$\Rightarrow u(x,t) = \sum_{n=0}^{\infty} A_n e^{-\left(\frac{n\pi}{L}\right)^2 Kt}\cos\left(\frac{n\pi x}{L}\right) + \sum_{n=1}^{\infty} B_n e^{-\left(\frac{n\pi}{L}\right)^2 Kt}$$

$$\frac{\partial u}{\partial x} = \sum_{n=0}^{\infty} -A_n e^{-\left(\frac{n\pi}{L}\right)^2 Kt}\sin\left(\frac{n\pi x}{L}\right)\left(\frac{n\pi}{L}\right) + \sum_{n=1}^{\infty} B_n e^{-\left(\frac{n\pi}{L}\right)^2 Kt}\cos\left(\frac{n\pi x}{L}\right)\left(\frac{n\pi}{L}\right)$$

$$\frac{\partial u(0,t)}{\partial x} = 0 = \sum_{n=1}^{\infty} B_n e^{-\left(\frac{n\pi}{L}\right)^2 Kt}\left(\frac{n\pi}{L}\right) \Longrightarrow B_n = 0$$

$\longrightarrow$

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$$\Rightarrow u(x,t) = \sum_{n=0}^{\infty} A_n e^{-\left(\frac{n\pi}{L}\right)^2 kt} \cos\left(\frac{n\pi x}{L}\right)$$

$$u(x,0) = f(x) = \sum_{n=0}^{\infty} A_n \cos\left(\frac{n\pi x}{L}\right)$$

$$\int_0^L \cos\left(\frac{m\pi x}{L}\right) f(x)\, dx = \int_0^L A_n \cos\left(\frac{n\pi x}{L}\right) \cos\left(\frac{m\pi x}{L}\right) dx$$

$$\Longrightarrow \qquad A_n = \frac{2}{L} \int_0^L f(x) \cos\left(\frac{n\pi x}{L}\right) dx$$

$$A_0 = \frac{1}{L} \int_0^L f(x)\, dx .$$