



Supplement to A Conjecture in Addition Chains Related to Scholz's Conjecture

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Source: *Mathematics of Computation*, Vol. 61, No. 203, Special Issue Dedicated to Derrick Henry Lehmer, (Jul., 1993), pp. S1-S6

Published by: American Mathematical Society

Stable URL: <http://www.jstor.org/stable/2152971>

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Supplement to
A CONJECTURE IN ADDITION CHAINS
RELATED TO SCHOLZ'S CONJECTURE

WALTER AIELLO AND M. V. SUBBARAO

Appendix

Methods used for construction of short addition chains for
 $2^n - 1$, n going from 2 to 128

NUM = $2^{**} 2 - 1$
 $l(n) = 1$ $S(\text{NUM}) = 2$
Methods used to form chain:
TWO

NUM = $2^{**} 9 - 1$
 $l(n) = 4$ $S(\text{NUM}) = 12$
Methods used to form chain:
ONE, THREE

NUM = $2^{**} 3 - 1$
 $l(n) = 2$ $S(\text{NUM}) = 4$
Methods used to form chain:
ONE

NUM = $2^{**} 10 - 1$
 $l(n) = 4$ $S(\text{NUM}) = 13$
Methods used to form chain:
ONE, TWO

NUM = $2^{**} 4 - 1$
 $l(n) = 2$ $S(\text{NUM}) = 5$
Methods used to form chain:
TWO

NUM = $2^{**} 11 - 1$
 $l(n) = 5$ $S(\text{NUM}) = 15$
Methods used to form chain:
ONE, TWO

NUM = $2^{**} 5 - 1$
 $l(n) = 3$ $S(\text{NUM}) = 7$
Methods used to form chain:
ONE, TWO

NUM = $2^{**} 12 - 1$
 $l(n) = 4$ $S(\text{NUM}) = 15$
Methods used to form chain:
ONE, TWO

NUM = $2^{**} 6 - 1$
 $l(n) = 3$ $S(\text{NUM}) = 8$
Methods used to form chain:
ONE, TWO

NUM = $2^{**} 13 - 1$
 $l(n) = 5$ $S(\text{NUM}) = 17$
Methods used to form chain:
ONE, TWO

NUM = $2^{**} 7 - 1$
 $l(n) = 4$ $S(\text{NUM}) = 10$
Methods used to form chain:
ONE

NUM = $2^{**} 14 - 1$
 $l(n) = 5$ $S(\text{NUM}) = 18$
Methods used to form chain:
ONE, TWO

NUM = $2^{**} 8 - 1$
 $l(n) = 3$ $S(\text{NUM}) = 10$
Methods used to form chain:
TWO

NUM = $2^{**} 15 - 1$
 $l(n) = 5$ $S(\text{NUM}) = 19$
Methods used to form chain:
ONE, TWO, THREE

NUM = 2** 16 - 1 l(n) = 4 S(NUM) = 19 Methods used to form chain: TWO	NUM = 2** 24 - 1 l(n) = 5 S(NUM) = 28 Methods used to form chain: ONE, TWO	NUM = 2** 32 - 1 l(n) = 5 S(NUM) = 36 Methods used to form chain: TWO	NUM = 2** 40 - 1 l(n) = 6 S(NUM) = 45 Methods used to form chain: ONE, TWO
NUM = 2** 17 - 1 l(n) = 5 S(NUM) = 21 Methods used to form chain: ONE, TWO	NUM = 2** 25 - 1 l(n) = 6 S(NUM) = 30 Methods used to form chain: ONE, TWO, FIVE	NUM = 2** 33 - 1 l(n) = 6 S(NUM) = 38 Methods used to form chain: TWO, Method for N = 33	NUM = 2** 41 - 1 l(n) = 7 S(NUM) = 47 Methods used to form chain: ONE, TWO
NUM = 2** 18 - 1 l(n) = 5 S(NUM) = 22 Methods used to form chain: ONE, TWO, THREE	NUM = 2** 26 - 1 l(n) = 6 S(NUM) = 31 Methods used to form chain: ONE, TWO	NUM = 2** 34 - 1 l(n) = 6 S(NUM) = 39 Methods used to form chain: ONE, TWO	NUM = 2** 42 - 1 l(n) = 7 S(NUM) = 48 Methods used to form chain: ONE, TWO, THREE
NUM = 2** 19 - 1 l(n) = 6 S(NUM) = 24 Methods used to form chain: ONE, THREE	NUM = 2** 27 - 1 l(n) = 6 S(NUM) = 32 Methods used to form chain: ONE, THREE	NUM = 2** 35 - 1 l(n) = 7 S(NUM) = 41 Methods used to form chain: ONE, TWO, SIX	NUM = 2** 43 - 1 l(n) = 7 S(NUM) = 49 Methods used to form chain: ONE, TWO, FOUR
NUM = 2** 20 - 1 l(n) = 5 S(NUM) = 24 Methods used to form chain: ONE, TWO	NUM = 2** 28 - 1 l(n) = 6 S(NUM) = 33 Methods used to form chain: ONE, TWO	NUM = 2** 36 - 1 l(n) = 6 S(NUM) = 41 Methods used to form chain: ONE, TWO, THREE	NUM = 2** 44 - 1 l(n) = 7 S(NUM) = 50 Methods used to form chain: ONE, TWO
NUM = 2** 21 - 1 l(n) = 6 S(NUM) = 26 Methods used to form chain: ONE, THREE	NUM = 2** 29 - 1 l(n) = 7 S(NUM) = 35 Methods used to form chain: ONE, TWO, FOUR	NUM = 2** 37 - 1 l(n) = 7 S(NUM) = 43 Methods used to form chain: ONE, TWO, THREE	NUM = 2** 45 - 1 l(n) = 7 S(NUM) = 51 Methods used to form chain: ONE, TWO, THREE
NUM = 2** 22 - 1 l(n) = 6 S(NUM) = 27 Methods used to form chain: ONE, TWO	NUM = 2** 30 - 1 l(n) = 6 S(NUM) = 35 Methods used to form chain: ONE, TWO, THREE	NUM = 2** 38 - 1 l(n) = 7 S(NUM) = 44 Methods used to form chain: ONE, TWO, THREE	NUM = 2** 46 - 1 l(n) = 7 S(NUM) = 52 Methods used to form chain: ONE, TWO, FOUR
NUM = 2** 23 - 1 l(n) = 6 S(NUM) = 28 Methods used to form chain: ONE, TWO, FOUR	NUM = 2** 31 - 1 l(n) = 7 S(NUM) = 37 Methods used to form chain: ONE, TWO, FOUR	NUM = 2** 39 - 1 l(n) = 7 S(NUM) = 45 Methods used to form chain: ONE, TWO, THREE	NUM = 2** 47 - 1 l(n) = 8 S(NUM) = 54 Methods used to form chain: ONE, TWO, FOUR

NUM = 2** 48 - 1
 l(n) = 6 S(NUM) = 53
 Methods used to form chain:
 ONE, TWO

NUM = 2** 49 - 1
 l(n) = 7 S(NUM) = 55
 Methods used to form chain:
 ONE, TWO

NUM = 2** 50 - 1
 l(n) = 7 S(NUM) = 56
 Methods used to form chain:
 ONE, TWO, FIVE

NUM = 2** 51 - 1
 l(n) = 7 S(NUM) = 57
 Methods used to form chain:
 ONE, TWO, THREE

NUM = 2** 52 - 1
 l(n) = 7 S(NUM) = 58
 Methods used to form chain:
 ONE, TWO

NUM = 2** 53 - 1
 l(n) = 8 S(NUM) = 60
 Methods used to form chain:
 ONE, TWO

NUM = 2** 54 - 1
 l(n) = 7 S(NUM) = 60
 Methods used to form chain:
 ONE, TWO, THREE

NUM = 2** 55 - 1
 l(n) = 8 S(NUM) = 62
 Methods used to form chain:
 ONE, TWO, FIVE

NUM = 2** 56 - 1
 l(n) = 7 S(NUM) = 62
 Methods used to form chain:
 ONE, TWO

NUM = 2** 57 - 1
 l(n) = 8 S(NUM) = 64
 Methods used to form chain:
 ONE, THREE

NUM = 2** 58 - 1
 l(n) = 8 S(NUM) = 65
 Methods used to form chain:
 ONE, TWO, FOUR

NUM = 2** 59 - 1
 l(n) = 8 S(NUM) = 66
 Methods used to form chain:
 ONE, TWO, FOUR

NUM = 2** 60 - 1
 l(n) = 7 S(NUM) = 66
 Methods used to form chain:
 ONE, TWO, THREE

NUM = 2** 61 - 1
 l(n) = 8 S(NUM) = 68
 Methods used to form chain:
 ONE, TWO, THREE

NUM = 2** 62 - 1
 l(n) = 8 S(NUM) = 69
 Methods used to form chain:
 ONE, TWO, FOUR

NUM = 2** 63 - 1
 l(n) = 8 S(NUM) = 70
 Methods used to form chain:
 ONE, THREE

NUM = 2** 64 - 1
 l(n) = 6 S(NUM) = 69
 Methods used to form chain:
 TWO

NUM = 2** 65 - 1
 l(n) = 7 S(NUM) = 71
 Methods used to form chain:
 ONE, TWO

NUM = 2** 66 - 1
 l(n) = 7 S(NUM) = 72
 Methods used to form chain:
 TWO, Method for N = 33

NUM = 2** 67 - 1
 l(n) = 8 S(NUM) = 74
 Methods used to form chain:
 ONE, TWO, Method for N = 33

NUM = 2** 68 - 1
 l(n) = 7 S(NUM) = 74
 Methods used to form chain:
 ONE, TWO

NUM = 2** 69 - 1
 l(n) = 8 S(NUM) = 76
 Methods used to form chain:
 ONE, TWO, THREE, FOUR

NUM = 2** 70 - 1
 l(n) = 8 S(NUM) = 77
 Methods used to form chain:
 ONE, TWO, SIX

NUM = 2** 71 - 1
 l(n) = 9 S(NUM) = 79
 Methods used to form chain:
 ONE, TWO, FOUR, Method N = 33

NUM = 2** 72 - 1
 l(n) = 7 S(NUM) = 78
 Methods used to form chain:
 ONE, TWO, THREE

NUM = 2** 73 - 1
 l(n) = 8 S(NUM) = 80
 Methods used to form chain:
 ONE, TWO, THREE, Meth for N = 33

NUM = 2** 74 - 1
 l(n) = 8 S(NUM) = 81
 Methods used to form chain:
 ONE, TWO, THREE

NUM = 2** 75 - 1
 l(n) = 8 S(NUM) = 82
 Methods used to form chain:
 ONE, TWO, THREE, FIVE

NUM = 2** 76 - 1
 l(n) = 8 S(NUM) = 83
 Methods used to form chain:
 ONE, TWO, THREE

NUM = 2** 77 - 1
 l(n) = 8 S(NUM) = 84
 Methods used to form chain:
 ONE, TWO, Method for N = 77

NUM = 2** 78 - 1
 l(n) = 8 S(NUM) = 85
 Methods used to form chain:
 ONE, TWO, THREE

NUM = 2** 79 - 1
 l(n) = 9 S(NUM) = 87
 Methods used to form chain:
 ONE, TWO, THREE

NUM = 2**80 - 1
 l(n) = 7 S(NUM) = 86
 Methods used to form chain:
 ONE, TWO

NUM = 2**81 - 1
 l(n) = 8 S(NUM) = 88
 Methods used to form chain:
 ONE, THREE

NUM = 2**82 - 1
 l(n) = 8 S(NUM) = 89
 Methods used to form chain:
 ONE, TWO

NUM = 2**83 - 1
 l(n) = 8 S(NUM) = 90
 Methods used to form chain:
 ONE, TWO, FOUR

NUM = 2**84 - 1
 l(n) = 8 S(NUM) = 91
 Methods used to form chain:
 ONE, TWO, THREE

NUM = 2**85 - 1
 l(n) = 8 S(NUM) = 92
 Methods used to form chain:
 ONE, TWO, FIVE

NUM = 2**86 - 1
 l(n) = 8 S(NUM) = 93
 Methods used to form chain:
 ONE, TWO, FOUR

NUM = 2**87 - 1
 l(n) = 9 S(NUM) = 95
 Methods used to form chain:
 ONE, TWO, THREE, FOUR

NUM = 2**88 - 1
 l(n) = 8 S(NUM) = 95
 Methods used to form chain:
 ONE, TWO

NUM = 2**89 - 1
 l(n) = 9 S(NUM) = 97
 Methods used to form chain:
 ONE, TWO, FOUR

NUM = 2**90 - 1
 l(n) = 8 S(NUM) = 97
 Methods used to form chain:
 ONE, TWO, THREE

NUM = 2**91 - 1
 l(n) = 9 S(NUM) = 99
 Methods used to form chain:
 ONE, TWO, THREE, SIX

NUM = 2**92 - 1
 l(n) = 8 S(NUM) = 99
 Methods used to form chain:
 ONE, TWO, FOUR

NUM = 2**93 - 1
 l(n) = 9 S(NUM) = 101
 Methods used to form chain:
 ONE, TWO, THREE, FOUR

NUM = 2**94 - 1
 l(n) = 9 S(NUM) = 102
 Methods used to form chain:
 ONE, TWO, FOUR

NUM = 2**95 - 1
 l(n) = 9 S(NUM) = 103
 Methods used to form chain:
 ONE, TWO, THREE, FIVE

NUM = 2**96 - 1
 l(n) = 7 S(NUM) = 102
 Methods used to form chain:
 ONE, TWO

NUM = 2**97 - 1
 l(n) = 8 S(NUM) = 104
 Methods used to form chain:
 ONE, TWO

NUM = 2**98 - 1
 l(n) = 8 S(NUM) = 105
 Methods used to form chain:
 ONE, TWO

NUM = 2**99 - 1
 l(n) = 8 S(NUM) = 106
 Methods used to form chain:
 TWO, THREE, Method for N = 33

NUM = 2**100 - 1
 l(n) = 8 S(NUM) = 107
 Methods used to form chain:
 ONE, TWO, FIVE

NUM = 2**101 - 1
 l(n) = 9 S(NUM) = 109
 Methods used to form chain:
 ONE, TWO

NUM = 2**102 - 1
 l(n) = 8 S(NUM) = 109
 Methods used to form chain:
 ONE, TWO, THREE

NUM = 2**103 - 1
 l(n) = 9 S(NUM) = 111
 Methods used to form chain:
 ONE, TWO, THREE

NUM = 2**104 - 1
 l(n) = 8 S(NUM) = 111
 Methods used to form chain:
 ONE, TWO

NUM = 2**105 - 1
 l(n) = 9 S(NUM) = 113
 Methods used to form chain:
 ONE, TWO, THREE, SIX

NUM = 2**106 - 1
 l(n) = 9 S(NUM) = 114
 Methods used to form chain:
 ONE, TWO

NUM = 2**107 - 1
 l(n) = 9 S(NUM) = 115
 Methods used to form chain:
 ONE, TWO, FOUR

NUM = 2**108 - 1
 l(n) = 8 S(NUM) = 115
 Methods used to form chain:
 ONE, TWO, THREE

NUM = 2**109 - 1
 l(n) = 9 S(NUM) = 117
 Methods used to form chain:
 ONE, TWO, THREE

NUM = 2**110 - 1
 l(n) = 9 S(NUM) = 118
 Methods used to form chain:
 ONE, TWO, FIVE

NUM = 2**111 - 1
 l(n) = 9 S(NUM) = 119
 Methods used to form chain:
 ONE, TWO, THREE

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NUM = 2**112 - 1
l(n) = 8 S(NUM) = 119
Methods used to form chain:
ONE, TWO

NUM = 2**113 - 1
l(n) = 9 S(NUM) = 121
Methods used to form chain:
ONE, TWO

NUM = 2**114 - 1
l(n) = 9 S(NUM) = 122
Methods used to form chain:
ONE, TWO, THREE

NUM = 2**115 - 1
l(n) = 9 S(NUM) = 123
Methods used to form chain:
ONE, TWO, FOUR

NUM = 2**116 - 1
l(n) = 9 S(NUM) = 124
Methods used to form chain:
ONE, TWO, FOUR

NUM = 2**117 - 1
l(n) = 9 S(NUM) = 125
Methods used to form chain:
ONE, TWO, THREE

NUM = 2**118 - 1
l(n) = 9 S(NUM) = 126
Methods used to form chain:
ONE, TWO, FOUR

NUM = 2**119 - 1
l(n) = 9 S(NUM) = 127
Methods used to form chain:
ONE, TWO, SIX

NUM = 2**120 - 1
l(n) = 8 S(NUM) = 127
Methods used to form chain:
ONE, TWO, THREE

NUM = 2**121 - 1
l(n) = 9 S(NUM) = 129
Methods used to form chain:
ONE, TWO, THREE

NUM = 2**122 - 1
l(n) = 9 S(NUM) = 130
Methods used to form chain:
ONE, TWO, THREE

NUM = 2**123 - 1
l(n) = 9 S(NUM) = 131
Methods used to form chain:
ONE, TWO, THREE

NUM = 2**124 - 1
l(n) = 9 S(NUM) = 132
Methods used to form chain:
ONE, TWO, FOUR

NUM = 2**125 - 1
l(n) = 9 S(NUM) = 133
Methods used to form chain:
ONE, TWO, THREE, FIVE

NUM = 2**126 - 1
l(n) = 9 S(NUM) = 134
Methods used to form chain:
ONE, TWO, THREE

NUM = 2**127 - 1
l(n) = 10 S(NUM) = 136
Methods used to form chain:
ONE, TWO, FOUR

NUM = 2**128 - 1
l(n) = 7 S(NUM) = 134
Methods used to form chain:
TWO
    
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NUM = 2**33 - 1
Chain length: 38
Scholz length: 38
Chain PASSES

Methods used to form chain:
Method TWO
SPECIAL METHOD for N = 33
+ 858934591
+ 4294567296
+ 4294567295
+ 4294501760
+ 2147450880
+ 1073725440
+ 536862720
+ 268431360
+ 134215680
+ 67107840
+ 33553920
+ 16776960
+ 8388480
+ 4194240
+ 2097120
+ 1048560
+ 524280
+ 262140
+ 131070
+ 65535
+ 65280
+ 32640
+ 16320
+ 8160
+ 4080
+ 2040
+ 1020
+ 510
+ 255
+ 240
+ 120
+ 60
+ 30
+ 15
+ 12
+ 6
+ 3
+ 2
+ 1
    
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NUM = 2** 77 - 1
Chain length: 84
Scholz length: 84
Chain PASSES

Methods used to form chain:
Method ONE
Method TWO
SPECIAL METHOD for N = 77
+ 151115727451828646838271
+ 4294934528
+ 2147467564
+ 1073733632
+ 536866816
+ 268433408
+ 134216704
+ 67108352
+ 33554176
+ 16777088
+ 8368544
+ 499472
+ 2057156
+ 5048568
+ 524284
+ 262142
+ 131071
+ 130560
+ 65280
+ 32640
+ 16320
+ 8160
+ 4080
+ 2040
+ 1020
+ 511
+ 510
+ 255
+ 240
+ 120
+ 60
+ 30
+ 15
+ 12
+ 6
+ 3
+ 2
+ 1

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