Our next conjecture is

2. *$Q(x)$ converges to $y_2(x)$ for all $x>1$.*

It is easy to prove by induction that if for any $x<0$, $R(x)\to z$, then $Q(x)\to\bar{z}$, the complex conjugate of z. Thus if conjecture 1 is correct, then we know that the values of $Q(x)$ for $-\frac{1}{4}<x<0$ are given not by y_2 but, surprisingly, by y_1. In the range $0<x<1$, the partial radicals associated with Q are complex for all $n\geqq 2$, whereas the corresponding $f_n(x)$ are real, and the induction proof cannot be carried over. Nevertheless, it would still appear that

3. *For all $0<x<1$, $Q(x)\to y_1(x)$.* One may also ask:

4. *What iterated radical, if any, converges to $y_2(x)$ for $-\frac{1}{4}<x<0$ and $0<x<1$?*

Finally, a broader objective would be:

5. *Develop procedures by which to investigate the convergence of more general iterated radicals, such as*

$$\sqrt{a_1+\sqrt{a_2+\sqrt{a_3+\cdots}}},$$

where the a_i are negative or complex. Herschfeld [1] remarks only that "Convergence questions appear to become very difficult in such cases."

Reference

1. Aaron Herschfeld, On infinite radicals, this MONTHLY, 42 (1935) 419.

ARE THERE AN INFINITY OF UNITARY PERFECT NUMBERS?

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A divisor d of n is said to be unitary if d and n/d are relatively prime. We shall denote the sum of the unitary divisors of n by $\sigma^*(n)$. Let n be called a unitary perfect number provided $\sigma^*(n)=2n$.

The inevitable question arises: are there an infinity of unitary perfect numbers? P. Erdös to whom the writer mentioned this problem in 1965, expressed the opinion that it might be a very difficult one, comparable to the problem of odd perfect numbers and he readily offered a prize of $10 for the first complete solution, to which the writer offers a supplemental prize of an equal amount.

It is trivial to show that there are no odd unitary perfect numbers. Suppose n is an even unitary perfect number and is of the form $n=2^a m$, where m is odd and has r distinct prime divisors. In a paper in 1965, L. J. Warren and the writer showed, by elementary methods, the following theorem:

THEOREM A.

(i) *If $r=1$, then $n=60$.*
(ii) *If $a=1$, then $n=6$ or 90.*
(iii) *If $a=2$, then $n=60$.*
(iv) *If $r=2$, then $n=60$ or 90.*
(v) *a cannot be 3, 4, 5 or 7.*
(vi) *r cannot be 3 or 5.*
(vii) *If $a=6$, then $n=87{,}360$.*
(viii) *If $r=4$, then $n=87{,}360$.*

A year ago, the writer together with three undergraduate students, T. J. Cook, R. S. Newberry and J. M. Weber, (participants in the Undergraduate Research Participation Program under National Science Foundation Grant No. GY 4599 to the University of Missouri) obtained further results in this direction including the following:

THEOREM B. *Let* $n=2^a m$ *be unitary perfect. With the same notation as in Theorem* A, (i) *it is not possible for* $a=8, 9, 10$; (ii) *it is not possible for* $r=6$.

The proof involves extensive and exhausting calculations using a desk calculator. The details are too long to be shown here.

These theorems can be used to show, for example, that after 87,360, there can be no unitary perfect number with less than 20 digits. The writer was recently informed, however, that a graduate student at the University of Tennessee, Mr. Charles R. Wall, found by accident another unitary perfect number, namely

$$2^{18}\cdot 3\cdot 5^4\cdot 7\cdot 11\cdot 13\cdot 19\cdot 37\cdot 79\cdot 109\cdot 157\cdot 313,$$

a number with 24 digits!

Therefore it seems rather unsafe to make the conjecture that there are only a finite number of unitary perfect numbers, but the writer is still inclined to make it!

Some other results involving these numbers will be given elsewhere.

References

1. M. V. Subbarao and L. J. Warren, Unitary perfect numbers, Canadian Math. Bull., 9 (1966) 147–153.
2. M. V. Subbarao, T. J. Cook, R. S. Newberry, and J. M. Weber, On unitary perfect numbers, (to appear).
3. C. R. Wall. (Private communication from L. J. Warren.)

CLASSROOM NOTES

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ISOLATION OF ZEROS IN THE SECOND ORDER LINEAR DIFFERENTIAL EQUATION

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Let $\psi(x)$ be a solution ($\psi\not\equiv 0$) to the second order linear differential equation

$$y'' + \alpha_1(x)y' + \alpha_2(x)y = \beta(x),$$

where $\alpha_1(x)$, $\alpha_2(x)$, and $\beta(x)$ are continuous on (a, b). When $\beta(x)\equiv 0$, it is well