Multiple equilibrium states and the abrupt transitions in a dynamical system of soil water interacting with vegetation

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1 In semi-arid areas, multiple equilibrium states of an ecosystem (e.g., grassland and desert) are found to coexist, and the transition from grassland to desert is often abrupt at the boundary. A simple ecosystem model is developed to provide the biophysical explanation of this phenomenon. The model has three variables: living biomass, wilted biomass, and soil wetness. The moisture index, which is the ratio of the annual precipitation to potential evaporation, is the only external climate driving force, and the key mechanism is the vegetation-soil interaction. It is found that the maintenance of a grassland requires a minimum moisture index, and the abrupt transition occurs when the moisture index is around this critical value. These results are robust within a wide range for most model parameters, suggesting that the model may be applicable to other temperate grasslands. The characteristics of the wilted biomass also strongly influence the ecosystem’s dynamics. INDEX TERMS: Oceanography: Biological and Chemical: Ecosystems, structure and dynamics; 4842 Oceanography: Biological and Chemical: Modeling; 1809 Hydrology: Desertification. Citation: Zeng X., S. S. P. Shen, X. Zeng, and R. E. Dickinson (2004), Multiple equilibrium states and the abrupt transitions in a dynamical system of soil water interacting with vegetation, Geophys. Res. Lett., 31, LXXXX, doi:10.1029/2003GL018910.

1. Introduction

2 The composition of an ecosystem is determined primarily by latitude (sunlight and temperature), altitude (temperature) and precipitation (soil moisture). Over the regions with favorable sunlight and temperature conditions, the so-called “moisture index”, i.e., the ratio of annual precipitation to potential evaporation, is the most important parameter. The vegetation distribution is usually continuous in space, but can also be discontinuous, even in regions with a smooth variation of the moisture index. For example, in Inner Mongolia, China, a typical steppe has ample vegetation, e.g., Stipa grandis and Stipa korevii, while the desert or semi-desert zone grows only sparse arid grass or shrubs, with a biomass that is about one to two orders of magnitude smaller than that in the steppe. Figure 1 indicates that the boundary between the grassland and desert is sharp and roughly along the line of a moisture index of 0.3. Such transitions between these two ecosystems can occur in both spatial and temporal domains. Over the Saharan and Sahel regions of Africa, the temporal evolution of an ecosystem has been investigated by using simple to intermediate-level models [Brovkin et al., 1998; Claussen et al., 1998; Zeng et al., 1999; Wang and Eltahir, 2000]. These models demonstrated that multiple equilibrium states could coexist in the biosphere-atmosphere system. Under certain conditions, subtle variations of climate could be strongly amplified by atmosphere-vegetation feedbacks, triggering an abrupt switch of the biosphere-atmosphere system from one equilibrium state to another.

3 In the study of the spatial variation of the ecosystem, the climate driven vegetation model of Zeng et al. [1994] and Zeng and Zeng [1996] demonstrated that, with the vegetation-soil interaction, the land system alone could possess multiple equilibrium states and display spatial discontinuity between grassland and desert. However, that model was of maximum simplification, consisting of only two variables, i.e., the total biomass and soil wetness. Therefore the difference in the functions of the living and wilted (dead) vegetation as well as the difference in the mechanisms of evaporation and transpiration could not be distinguished. This highly idealized model requires an unusually strong interaction between biomass and soil wetness to produce the abrupt change from grassland to desert. The current study improves the realism of that previous model through the introduction of a new variable: wilted biomass. With this third variable, the observed abrupt transition between grassland and desert in Inner Mongolia can be better interpreted. This new model can be used in stand-alone mode or coupled to climate models to explore the general behavior of the self-organization and vegetation-soil interaction in a grassland ecosystem.

2. The Model

Our new model of grassland considers a single vertical column of soil and one species of grass. It has three dimensionless state variables: the mass density of living leaves, $x$, the available soil wetness, $y$, and the mass density of wilted (dead) leaves, $z$. The actual values of these variables are described by $x' = xx^*$, $y' = yy^*$, and $z' = zz^*$. Copyright 2004 by the American Geophysical Union. 0094-8276/04/2003GL018910S05.00
respective, where dimensional constants $x$, $y$, and $z$ are the corresponding characteristic values. This model is built on the earlier idealized model of Zeng et al. [1994] and Zeng and Zeng [1996] but differs from it by more realistic biophysical considerations in two aspects: (a) the number of prognostic variables is increased from two to the current three so that the impact of wilted biomass on the vegetation-soil interaction can be explicitly considered; and (b) the surface evaporation and transpiration are computed separately in the current model (rather than mixed together as in the 2-variable model).

For simplicity, it is assumed that the living grass covers part of the soil surface, while the wilted grass is distributed uniformly over the soil surface. Precipitation, other atmospheric conditions, and the soil properties are all prescribed. The system is assumed to be horizontally homogeneous. Following the mass conservation law, the dynamics of the ecosystem can be written as

$$\frac{dx}{dt} = x^* \frac{dx}{dx} + F_1 = G(x,y) - D(x,y) - C(x), \quad (1)$$

$$\frac{dy}{dt} = y^* \frac{dy}{dy} + F_2 = P - E_0(x,y,z) - E_r(x,y) - R(x,y,z), \quad (2)$$

and

$$\frac{dz}{dt} = z^* \frac{dz}{dz} + F_3 = G_z(x,y) - D_z(z) - C_z(z), \quad (3)$$

where terms $G$, $D$, and $C$ are the growth (photosynthesis without plant respiration), wilting, and consumption (grazing) of the living leaves, $G_z$, $D_z$, and $C_z$ are the accumulation, decomposition, and consumption of the wilted grass, $P$ is the atmospheric precipitation (system 122 input), $E_r$ is the evaporation from the soil surface, $E_r$ is the transpiration, and $R$ is runoff. Note that, more accurately, $P$ is the through fall, i.e., precipitation minus intercepted water by live and wilted leaves. Intercepted precipitation is relatively small, and our sensitivity tests show that conclusions remain the same if this term is included.

Figure 1. Distribution of vegetation zone and moisture index in Inner Mongolia. In the desert or semi-desert zones (IV3; V3; VI1, 2) only sparse arid grass or shrubs grow, and the biomass is about one to two orders of magnitude smaller than that in the steppe regions (IV1, 2; V1, 2). The boundary between the grassland and desert is roughly along the line of a moisture index of 0.3. (This figure is reconstructed with permission from data and maps provided by G. Zhao of Inner Mongolian Agricultural University.)
Investigated by numerically integrating equations (1) to (3).

Between Ecosystems

Relative humidity, saturation vapor pressure, and stomatal coverage, and the growth and wilting of living biomass at the case of full wilting rate of the corresponding values over the vast grassland, the [11] by observational data in the Inner Mongolia grassland [13]. All these parameters and coefficients are determined where the runoff term considers not only the impact of precipitation, and soil wetness but also of living and wilted leaves as follows:

\[ R = \lambda e^{\sigma_1 (1 - e^{-\gamma_1 \mu})} + \sigma_2 (1 - e^{-\gamma_2 (1 - e^{-\gamma_3 \mu})}). \]

where \( \lambda \) and \( \kappa'' \) are coefficients.

[11] All these parameters and coefficients are determined by observational data in the Inner Mongolia grassland [Chinese Academy of Sciences, 1985; Jiang, 1988] and shown in Table 1. For example, \( \lambda^* \) and \( \lambda' \) are the average of the corresponding values over the vast grassland, the wilting rate \( \beta \) is subjected to the requirement of balancing the growth and wilting of living biomass at the case of full coverage, and \( \kappa_1 \) and \( \varphi_{12} \) are influenced by factors such as relative humidity, saturation vapor pressure, and stomatal resistance of leaves, etc.

3. Equilibrium States and the Abrupt Transition Between Ecosystems

[12] The quantitative behaviors of the ecosystem can be investigated by numerically integrating equations (1) to (3). The equilibrium states of the system are of most interest and can be determined by setting \( F_1 = F_2 = F_3 = 0 \). The stability of the equilibrium states can be determined by means of a 196 general mathematical method [e.g., see Shen, 1993, Section 1.2]. Briefly speaking, equations (1) to (3) are first linearly expanded around each equilibrium, and the eigenvalues of the corresponding Jacob matrix are calculated. If the real part each of the three eigenvalues is negative, the equilibrium is stable, and the system will return to the equilibrium state from any small perturbations. Otherwise, the equilibrium is unstable (a saddle), and a small perturbation will normally cause the system to diverge from this state. Each stable equilibrium state corresponds to a possible ecosystem, either grassland or desert, whereas the unstable state is not expected in nature.

[13] Figure 2 shows the equilibrium states under different values of the prescribed moisture index \( \mu \). When \( \mu \) is less than a critical value \( \mu_1 \), there is only one equilibrium state, and it is stable with no living grass; that is, the ecosystem is a desert. When \( \mu \) is larger than another critical value \( \mu_2 > \mu_1 \), there are two equilibrium states, one stable with a sufficient amount of living biomass, and another unstable with no grass; that is, the ecosystem is a grassland. For \( \mu_1 < \mu < \mu_2 \), there are three equilibrium states, one unstable, and two stable with one for grassland and another for desert. This implies that (a) a minimum moisture index (i.e., \( \mu_1 \)) is required to maintain a grassland; and (b) an abrupt transition between grassland and desert occurs around \( \mu_1 \), and the biomass increases continuously with \( \mu \) when \( \mu > \mu_1 \).

[14] The biophysical explanations of the ecosystem dynamics shown in Figure 2 are as follows. Qualitatively, the increase of soil wetness always promotes the growth of living biomass. The living vegetation not only extracts soil water through transpiration but also conserves it through reducing the surface evaporation, while the wilted leaves (as well as other non-photosynthesizing surfaces) provide the benefits of shading without the use of the soil water. This principle of “mulching” is well known to farmers and gardeners. For small enough precipitation, the balance of precipitation with evaporation maintains soil water at too low a level for vegetation to succeed, so there is only one solution of bare soil (desert). With enough precipitation, this solution is unstable, and a solution with finite vegetation, i.e., grassland, is possible. Because of the stabilizing effects of the shading by living and wilted vegetation, the grassland can maintain its stability over a range of precipitation for which the bare soil solution is also stable.

[15] To check the robustness of the above results, sensitivity tests have been done by changing the value of a particular coefficient. For most of the coefficients, the 243

### Table 1. Values of Parameters and Coefficients Adjusted to the Inner Mongolia Grassland

<table>
<thead>
<tr>
<th>Dimensional Parameters</th>
<th>Dimensionless Coefficients</th>
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</thead>
<tbody>
<tr>
<td>( x^* )</td>
<td>0.1 kg m(^{-2} )</td>
</tr>
<tr>
<td>( y^* )</td>
<td>240 mm</td>
</tr>
<tr>
<td>( z^* )</td>
<td>0.1 kg m(^{-2} )</td>
</tr>
<tr>
<td>( \alpha^* )</td>
<td>0.4 kg m(^{-2} ) yr(^{-1} )</td>
</tr>
<tr>
<td>( e^* )</td>
<td>1000 mm yr(^{-1} )</td>
</tr>
<tr>
<td>( \beta )</td>
<td>0.6</td>
</tr>
<tr>
<td>( \lambda )</td>
<td>0.15</td>
</tr>
<tr>
<td>( \kappa_1 )</td>
<td>0.4</td>
</tr>
<tr>
<td>( \kappa_2 )</td>
<td>1.0</td>
</tr>
<tr>
<td>( \kappa'' )</td>
<td>0.7</td>
</tr>
<tr>
<td>( \varphi_{12} )</td>
<td>1.0</td>
</tr>
<tr>
<td>( \varphi_{12} )</td>
<td>1.0</td>
</tr>
</tbody>
</table>

The equilibrium states of (a) living biomass \( x \), (b) soil wetness \( y \), and (c) wilted biomass \( z \), as a function of the moisture index \( \mu \). The two critical values are indicated as \( \mu_1 \) and \( \mu_2 \). Solid and dash lines refer to stable and unstable equilibrium states respectively. Values of parameters and coefficients are shown in Table 1.
Figure 3. Sensitivity of the equilibrium states to the coefficient $k_1$ (shading effect of living biomass), $\varphi_2$ (the potential transpiration rate), and $\varepsilon_3$ (the exponential attenuated coefficient of wilted biomass shading). From left to right, (a) $k_1 = 0.6, 0.4, 0.2, \text{ and } 0$; (b) $\varphi_2 = 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, \text{ and } 1.0$; (c) $\varepsilon_3 = 1.0, 0.8, 0.6, 0.45, 0.3, 0.15 \text{ and } 0$. All other coefficients are kept the same as those in Figure 2.

4. Conclusions and Further Discussion

[17] A simple 3-variable ecosystem model is developed to study grassland ecosystem. The three variables are living biomass, wilted biomass, and soil wetness. It is found that multiple equilibrium states of ecosystems coexist in semi-arid areas, and the minimum moisture index for the existence of grassland is around 0.3 which is in agreement with observations in the Inner Mongolia grassland. The sufficient condition for the existence of grassland is that the moisture index $\mu$ be greater than another critical value $\mu_2 > \mu_1$. For $\mu_1 < \mu < \mu_2$, there exist two stable equilibrium states, one for grassland and another for desert. In other words, the change of biomass is abrupt even when $\mu$ changes smoothly between $\mu_1$ and $\mu_2$.

[18] The key mechanism of the system analyzed is the shading of otherwise sunlight soil by living and wilted vegetation. The introduction of wilted biomass as a prognostic variable is especially important. In the ecosystem with soil surface shaded by plenty of wilted biomass, soil evaporation is reduced significantly, and hence the soil wetness is conserved. This shading may be important for the occurrence of vegetation in semi-arid areas. Many suggestions have been made in the past to improve desertified landscapes through regeneration based on the belief that the introduction of the vegetation would increase precipitation enough to lead to a stable regime with more vegetation and precipitation. Such efforts may seem fruitless because of the likely smallness of the increase of precipitation. However, the mechanisms considered in this study suggest that the presence of vegetation alone may stabilize soil moisture at a high enough level to maintain the vegetation. This possibility should be further studied with accurate ecological data for semi-arid areas.

[19] The variation of $\mu$ can be discussed in both the spatial and temporal domains, and the former has been emphasized so far. In the spatial domain, the behavior of the multiple equilibrium states should be a concern in grassland maintenance in semi-arid areas. For the zone of $\mu_1 < \mu < \mu_2$, a certain amount of living and wilted biomass, determined by the corresponding unstable equilibrium state, is required to form the grassland, and an overuse of the living or wilted grass of the existing grassland might eventually lead to the stable equilibrium state of a desert. The range of parameter space over which these equilibriums can coexist may be increased by positive feedbacks of evapotranspiration on the precipitation [e.g., Wang and Eltahir, 2000]. The precipitation forcing and various coefficients may be taken to vary periodically and stochastically in time. With such variation, the simple system we consider may be viewed as a characterization of a complete nonlinear climate system with interactive vegetation. As such it provides insight into “attractor basins”, i.e., states of the complete system such as drought (or desert) regimes that maintain their stability for long periods of time.

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References


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