1. Let $S$ be the surface of the ball centered at $(0,0,0)$ with radius $r > 0$. Compute
\[ \int_{S} x^3 \, dy \wedge dz + y^3 \, dz \wedge dx + z^3 \, dx \wedge dy. \]

2. Let $V$ be a normal domain with boundary $S$ such that $N \neq 0$ on $S$ throughout, and let $f$ and $g$ be $\mathbb{R}$-valued $C^2$-functions on an open set containing $V$.

(a) Prove Green’s First Formula:
\[ \int_{V} (\nabla f) \cdot (\nabla g) + \int_{V} f \Delta g = \int_{S} f D_{n} g \, d\sigma. \]

(b) Prove Green’s Second Formula:
\[ \int_{V} (f \Delta g - g \Delta f) = \int_{S} (f D_{n} g - g D_{n} f) \, d\sigma. \]

(Hint for (a): Apply Gauß’ Theorem to the vector field $f \nabla g$.)

3. Let $\emptyset \neq U \subset \mathbb{R}^3$ be open, and suppose that $f \in C^2(U, \mathbb{R})$ is harmonic, i.e., satisfies $\Delta f = 0$. Let $V \subset U$, $S$ and $n$ be as in the previous problem. Show that
\[ \int_{S} D_{n} f \, d\sigma = 0 \quad \text{and} \quad \int_{S} f D_{n} f \, d\sigma = \int_{V} \|\nabla f\|^2. \]

4. Determine whether or not each of the following series converges or converges absolutely.

(a) $\sum_{n=1}^{\infty} \frac{1}{\cos(n) + \pi}$;

(b) $\sum_{n=1}^{\infty} \frac{\cos(\pi n)}{\sqrt{n + 4}}$;

(c) $\sum_{n=1}^{\infty} \frac{(-1)^n(n + 1)}{n^3}$. 
5. Prove or give a counterexample to the following generalization of the Alternating Series Test:

Let \((a_n)_{n=1}^{\infty}\) be a sequence of non-negative reals such that \(\lim_{n \to \infty} a_n = 0\).

Then \(\sum_{n=1}^{\infty} (-1)^{n-1} a_n\) converges.

\((\text{Hint: Try } a_n := \left| \frac{1}{n} - \frac{(-1)^n}{\sqrt{n}} \right|.)\)

6*. Let \((a_n)_{n=1}^{\infty}\) be a decreasing sequence of non-negative real numbers. Show that \(\sum_{n=1}^{\infty} a_n\) converges if and only if \(\sum_{n=1}^{\infty} 2^n a_{2^n}\) converges.

What can you conclude about the convergence of \(\sum_{n=1}^{\infty} \frac{1}{n^p}\) for \(p \in \mathbb{R}\)?

Due Monday, March 19, 2018, at 10:00 a.m.; no late assignments.