1. Let \( I \subset \mathbb{R}^N \) be a compact interval. Show that \( \partial I \) has content zero.

2. Let \( I \) be a compact interval, and let \( f = (f_1, \ldots, f_M) : I \to \mathbb{R}^M \). Show that \( f \) is Riemann integrable if and only if \( f_j : I \to \mathbb{R} \) is Riemann integrable for each \( j = 1, \ldots, M \) and that, in this case,

\[
\int_I f = \left( \int_I f_1, \ldots, \int_I f_M \right)
\]

holds.

3. Let \( I \subset \mathbb{R}^N \) be a compact interval, and let \( f : I \to \mathbb{R}^M \) be Riemann integrable. Show that \( f \) is bounded.

4. Let \( \emptyset \neq D \subset \mathbb{R}^N \) be bounded, and let \( f, g : D \to \mathbb{R} \) be Riemann-integrable. Show that \( fg : D \to \mathbb{R} \) is Riemann-integrable.

Do we have

\[
\int_D fg = \left( \int_D f \right) \left( \int_D g \right)
\]

(\textit{Hint}: First treat the case where \( f = g \), treat the general case by observing that \( fg = \frac{1}{2}((f + g)^2 - f^2 - g^2) \).)

5. Let \( \emptyset \neq U \subset \mathbb{R}^N \) be open with content, and let \( f : U \to [0, \infty) \) be bounded and continuous such that \( \int_U f = 0 \). Show that \( f \equiv 0 \) on \( U \).

6*. The function

\[ f : [0, 1] \times [0, 1] \to \mathbb{R}, \quad (x, y) \mapsto xy \]

is continuous and thus Riemann integrable. Evaluate \( \int_{[0, 1] \times [0, 1]} f \) using only the definition of the Riemann integral, i.e., in particular, without use of Fubini’s Theorem.

Due Monday, November 20, 2017, at 10:00 a.m.; no late assignments.