1. Let

\[ f : \mathbb{R}^2 \to \mathbb{R}, \quad (x, y) \mapsto \begin{cases} \frac{xy^3}{x^2+y^2}, & (x, y) \neq (0, 0), \\ 0, & (x, y) = (0, 0). \end{cases} \]

Show that:

(a) \( f \) is continuous at \((0, 0)\);

(b) for each \( v \in \mathbb{R}^2 \) with \( ||v|| = 1 \), the directional derivative \( D_v f(0, 0) \) exists and equals 0;

(c) \( f \) is not totally differentiable at \((0, 0)\).

(Hint for (c): Assume towards a contradiction that \( f \) is totally differentiable at \((0, 0)\), and compute the first derivative of \( \mathbb{R} \ni t \mapsto f(t^2, t) \) at 0 first directly and then using the chain rule. What do you observe?)

2. Let \( x, y \in \mathbb{R} \). Show that there is \( \theta \in [0, 1] \) such that

\[ \sin(x + y) = x + y - \frac{1}{2}(x^2 + 2xy + y^2)\sin(\theta(x + y)). \]

3. Determine and classify the stationary points of

\[ f : \mathbb{R}^2 \to \mathbb{R}, \quad (x, y) \mapsto (x^2 + 2y^2)e^{-(x^2+y^2)}. \]

If \( f \) has a local extremum at a stationary point, determine the nature of this extremum and evaluate \( f \) there.

4. Let \( c_1, \ldots, c_p \in \mathbb{R}^N \). For which \( x \in \mathbb{R}^N \) does \( \sum_{j=1}^{p} ||x - c_j||^2 \) become minimal?

5. Let \( (x_n)_{n=1}^{\infty} \) be a convergent sequence in \( \mathbb{R}^N \) with limit \( x \). Show that \( \{x_n : n \in \mathbb{N}\} \cup \{x\} \) has content zero.

6*. Determine the minimum and the maximum of

\[ f : D \to \mathbb{R}, \quad (x, y) \mapsto \sin x + \sin y + \sin(x + y), \]

where \( D := \{(x, y) \in \mathbb{R}^2 : 0 \leq x, y \leq \frac{\pi}{2}\} \), and all points of \( D \) where they are attained.

Due Monday, November 6, 2017, at 10:00 a.m.; no late assignments.