1. Let $a, b > 0$. Determine the area of the ellipse

$$E := \{ (x, y) \in \mathbb{R}^2 : \frac{x^2}{a^2} + \frac{y^2}{b^2} \leq 1 \}.$$

2. Let $D \subset \mathbb{R}^3$ be the region in the first octant, i.e., with $x, y, z \geq 0$, which is bounded by the cylinder given by $x^2 + y^2 = 16$ and the plane given by $z = 3$. Evaluate

$$\int_D xyz.$$

3. Let $R > 0$, and let

$$B := \{ (x, y, z) \in \mathbb{R}^3 : x^2 + y^2 + z^2 \leq R^2 \}$$

and

$$C := \{ (x, y, z) \in \mathbb{R}^3 : x^2 - Rx + y^2 \leq 0, z \geq 0 \}.$$

Determine $\mu(B \cap C)$.

4. Let $R > 0$, and define, for $0 \rho r < R$,

$$A_{\rho, R} := \{ (x, y, z) \in \mathbb{R}^3 : \rho^2 \leq x^2 + y^2 + z^2 \leq R^2 \}.$$

Determine

$$\lim_{\rho \to 0} \int_{A_{\rho, R}} \frac{1}{\sqrt{x^2 + y^2 + z^2}}.$$

5. Let $D$ in spherical coordinates be given as the solid lying between the spheres given by $r = 2$ and $r = 4$, above the $xy$-plane and below the cone given by the angle $\theta = \frac{\pi}{3}$. Evaluate the integral $\int_D xyz$.

6*. Let $D \subset \mathbb{R}^2$ be the trapeze with vertices $(1, 0), (2, 0), (0, -2)$, and $(0, -1)$. Evaluate

$$\int_D \exp \left( \frac{x+y}{x-y} \right).$$

(Hint: Consider $\phi : \mathbb{R}^2 \to \mathbb{R}^2$, $(u, v) \mapsto \left( \frac{1}{2}(u + v), \frac{1}{2}(u - v) \right)$

and apply Change of Variables.)

Due Monday, December 4, 2017, at 10:00 a.m.; no late assignments.