Final Review Info

The Review will be held on **Dec. 20th**

at **2:00** in **ETLE 1-001**

The Review is free, however donations for EWB will be accepted.

The Review will be based on the attached questions, taken from recent finals

Multiple Choice Questions

Mark your answers ***on the official answer sheet on the last page*** and ***detach it***. It will be collected ***after 90 minutes***.

1. Let $\mathbf{F} = e^y\mathbf{i} + xe^y\mathbf{j} + (z+1)e^z\mathbf{k}$ and C be the path with $\mathbf{r}(t) = t\sin^3(\frac{1}{2}\pi t)\,\mathbf{i} + t^2\sin^5(\frac{1}{2}\pi t)\,\mathbf{j} + t^{99}\mathbf{k}$ for $0 \le t \le 1$. The value of $\int_C \mathbf{F}\cdot d\mathbf{r}$ is

   (a) 0 (b) $2e$ (c) $4e$ (d) $-2e$ (e) $-4e$.

2. If the mass of a solid is given by

$$\int_{x=0}^{1}\int_{y=0}^{\sqrt{1-x^2}}\int_{z=\sqrt{x^2+y^2}}^{\sqrt{2-x^2-y^2}} \sqrt{x^2+y^2+z^2}\,dz\,dy\,dx\,,$$

   then this mass equals

   (a) $\frac{2-\sqrt{2}}{4}\pi$ (b) $\frac{2+\sqrt{2}}{4}\pi$ (c) $\frac{2+\sqrt{2}}{2}\pi$ (d) $\frac{2-\sqrt{2}}{2}\pi$ (e) $(2+\sqrt{2})\pi$.

3. The value of $\int_0^4\int_{-\sqrt{4x-x^2}}^{0} \frac{x}{2}\,dy\,dx$ equals

   (a) π (b) 2π (c) 4π (d) 8π (e) 16π.

4. Let $g = \operatorname{div}(\mathbf{F} + \operatorname{curl}\mathbf{F})$ where $\mathbf{F} = -xy\,\mathbf{i} + xz^2\ln(y^2+1)\mathbf{j} + e^{xyz}\mathbf{k}$. The value of a for which $g = 0$ at $(a, 2, 0)$ is

   (a) -2 (b) -1 (c) 1 (d) 2 (e) does not exist.

5. Let $\mathbf{F} = e^x\sin y\,\mathbf{i} + (e^x\cos y + x)\,\mathbf{j}$. If C denotes the boundary of the region between the graphs of $y = x^2$ and $y = 1$, traversed counter-clockwise, then $\int_C \mathbf{F}\cdot d\mathbf{r}$ equals

   (a) $-\frac{4}{3}$ (b) $\frac{4}{3}$ (c) $-\frac{2}{3}$ (d) $\frac{2}{3}$ (e) 0.

6. The area of the region enclosed by the curve $r = 4 + 3\cos\theta$ is

   (a) $\frac{31\pi}{2}$ (b) $\frac{21\pi}{2}$ (c) $\frac{41\pi}{2}$ (d) $\frac{51\pi}{2}$ (e) $\frac{11\pi}{2}$.

Long Answer, Show all the work

1. A metal cable lies along the semi-circle $x^2 + z^2 = 1$, $z \geq 0$ in the xz-plane. Find the centre of mass $(\overline{x}, \overline{z})$ of the cable if the density ρ at the point (x, z) of the cable is given by $\rho(x, z) = 2 - x^2$.

2. Verify the Divergence Theorem for the vector field $\mathbf{F} = 2x\,\mathbf{i} + 3y\,\mathbf{j} + 4z\,\mathbf{k}$ on the region bounded by $x^2 + z^2 = 1$, above the plane $z = 0$, and between the planes $y = -2$ and $y = 2$.

3. Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F} = \cos x\,\mathbf{i} + z(x^2 + y^2)^{3/2}\mathbf{j} + yz^3\,\mathbf{k}$ and C is the simple closed curve given by $\mathbf{r}(t) = 2\sin t\cos t\,\mathbf{i} + 2\sin^2 t\,\mathbf{j} + 2\sin t\,\mathbf{k}$ with $0 \leq t \leq \pi$.

Hint: C is the intersection of the cylinder $x^2 + y^2 = 2y$ and the cone $z = \sqrt{x^2 + y^2}$.

Multiple Choice Questions

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3.

1. The iterated integral $\int_0^8 \int_{y^{\frac{1}{3}}}^2 e^{x^4}\,dx\,dy$ equals

   (a) $\frac{1}{2}e^8$ (b) $\frac{1}{2}(e^8 - 1)$ (c) $\frac{1}{4}(e^{16} + 1)$ (d) $\frac{1}{4}(e^{16} - 1)$ (e) $\frac{1}{4}e^{16}$

2. A lamina occupies the region inside $x^2 + y^2 = 2y$ but outside $x^2 + y^2 = 1$. If the density at any point is inversely proportional to its distance from the origin, with constant of proportionality K, then the y- coordinate of the center of mass equals:

   (a) 0 (b) $\frac{\sqrt{3}}{2(\sqrt{3}-\frac{\pi}{3})}$ (c) $\frac{\sqrt{3}}{\sqrt{3}-\frac{\pi}{3}}$ (d) $\frac{3\sqrt{3}}{2(3\sqrt{3}+\pi)}$ (e) $\frac{3}{3+\frac{\pi}{\sqrt{3}}}$

3. For every differentiable function $f = f(x, y, z)$ and differentiable 3−dimensional vector field $\vec{F} = \vec{F}(x, y, z)$, the vector field Curl$(f\vec{F})$ equals

   (a) $f\text{curl}(\vec{F}) + \vec{\nabla} f \times \vec{F}$ (b) $(\vec{\nabla} f \cdot \vec{F})\vec{\nabla} f$ (c) $\text{div}(\vec{F})\vec{\nabla} f$ (d) $(\vec{\nabla} f \cdot \vec{F})\vec{F}$
   (e) $f\text{curl}(\vec{F}) - (\vec{\nabla} f) \times \vec{F}$

4. If C is the path given by $\vec{r}(t) = \cos^{50}(\frac{\pi}{2}t)\,\vec{\imath} + \sin^{100}(\pi t)\,\vec{\jmath} + t^{1000}\,\vec{k}$, $0 \le t \le 1$ and

   $$\vec{F}(x, y, z) = (ye^{xy} + z\cos(xz) + 2x)\,\vec{\imath} + (y + xe^{xy})\,\vec{\jmath} + (x\cos(xz))\,\vec{k},$$

   then $\int_C \vec{F} \cdot d\vec{r}$ equals

   (a) 0 (b) 1 (c) −1 (d) $2e$ (e) $-2e$

5. A thin wire is bent in the shape of a semicircle $x^2 + y^2 = 4$, $x \ge 0$. If the density is a constant k, then the x- coordinate of the center of mass is:

   (a) 0 (b) $\frac{1}{\pi}$ (c) $\frac{2}{\pi}$ (d) $\frac{3}{\pi}$ (e) $\frac{4}{\pi}$

6. The volume of the solid region inside both the sphere $x^2 + y^2 + z^2 = 6$ and the paraboloid $z = x^2 + y^2$ equals

   (a) $2\pi(2\sqrt{6} - 9)$ (b) $\frac{2\pi}{3}(6\sqrt{6} - 11)$ (c) $2\pi(6\sqrt{6} - 11)$
   (d) $\frac{2\pi}{3}(2\sqrt{6} - 9)$ (e) $\frac{2\pi}{3}(2\sqrt{6} - 11)$

Long Answer Questions

You must show all your work.

1. Evaluate the surface integral $\iint_S xy\,dS$, where S is the portion of the cone $z = \sqrt{x^2+y^2}$ that lies within the first octant and inside the cylinder $x^2+y^2=1$.

2. Verify Stokes' Theorem for $\vec{F}(x,y,z) = y^2\,\vec{\imath} + x\,\vec{\jmath} + z^2\,\vec{k}$, and surface S given by the part of the paraboloid $z = x^2+y^2$ that lies below the plane $z=1$, oriented upward.

3. Use the Divergence Theorem to find the upward flux of

$$\vec{F}(x,y,z) = (x+y^2)\,\vec{\imath} + (3x^2y + y^3 - x^3)\,\vec{\jmath} + (z+1)\,\vec{k}$$

through the surface S given by the part of the cone $z = 2(1-\sqrt{x^2+y^2})$ that lies above the xy- plane.
HINT: Consider also a surface S_1 such that S and S_1 together enclose a solid.