LA Exam 3 - MC. (Yellow)

1. $$3x^2 + 3z^2\frac{\partial z}{\partial x} + 6xy \cdot 2z\frac{\partial z}{\partial x} + 6yz^2 = 0$$

$\therefore$ at $(0,1,1)$

$$3\frac{\partial z}{\partial x} + 6 = 0 \implies z = -2 \qquad \boxed{e}$$

2. $$\Delta f \simeq df = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy = \left.\frac{1}{\sqrt{21-x^2-y^2}}(-x)\,dx + \frac{1}{\sqrt{21-x^2-y^2}}(-y)\,dy\right|_{(2,1)}$$

$$= \frac{-2}{4}dx - \frac{1}{4}dy = -\frac{1}{2}dx - \frac{1}{4}dy$$

$$\therefore f(2.02, .92) = f(2,1) - \frac{1}{2}(.02) - \frac{1}{4}(-.08)$$

$$= 4 - .01 + .02 = 4.01 \qquad \boxed{d}$$

3. $$0 \le \frac{x^2}{x^2+y^2} \le 1 \qquad x\cos(y) \to 1 \cdot 0 = 0 \quad \text{as } (x,y) \to (0,0)$$

$\therefore$ by Sq. Theorem $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^3\cos(y)}{x^2+y^2} = 0$ $\boxed{b}$

4 $$\vec{\nabla} f = (1 + e^y\cos(xz)z)\vec{i} + (1 + e^y\sin(xz))\vec{j} + (1 + e^y\cos(xz)x)\vec{k}$$

$$\vec{\nabla} f\Big|_{(1,2,0)} = \vec{i} + \vec{j} + (1+e^2)\vec{k}$$

$$\therefore \text{direct. der.} = \left[\vec{i} + \vec{j} + (1+e^2)\vec{k}\right] \cdot \frac{\left[3\vec{i} - \vec{j} + c\vec{k}\right]}{\sqrt{9+1+c^2}} = 0$$

so $3 - 1 + c(1+e^2) = 0$ and $c = \dfrac{-2}{1+e^2}$ $\boxed{a}$

(LA1) YELLOW

(a) Let $F(x,y,z) = \sqrt{x} + \sqrt{2y} + \sqrt{z}$

The Normal vector to S at P is

$$\vec{N} = \nabla F(P) = \left\langle \frac{1}{2\sqrt{x_0}}, \frac{1}{\sqrt{2y_0}}, \frac{1}{2\sqrt{z_0}} \right\rangle$$

The EQ. of tangent plane to S at P

$$\frac{1}{2\sqrt{x_0}}(x-x_0) + \frac{1}{\sqrt{2y_0}}(y-y_0) + \frac{1}{2\sqrt{z_0}}(z-z_0) = 0$$

OR $$\frac{x}{2\sqrt{x_0}} + \frac{y}{\sqrt{2y_0}} + \frac{z}{2\sqrt{z_0}} = \underbrace{\frac{\sqrt{x_0}}{2} + \frac{\sqrt{2y_0}}{2} + \frac{\sqrt{z_0}}{2}}_{=\frac{\sqrt{3c}}{2}}$$

(b) $y = z = 0 \Rightarrow x_1 = \sqrt{x_0}\sqrt{3c}$

$x = z = 0 \Rightarrow 2y_1 = \sqrt{2y_0}\sqrt{3c}$

$x = y = 0 \Rightarrow z_1 = \sqrt{z_0}\sqrt{3c}$

$\Rightarrow x_1 + 2y_1 + z_1 = \sqrt{3c}\cdot\sqrt{3c} = 3c$

LA2 **Problem** (yellow). Find the maximum and minimum values of $f(x,y,z) = x^2 + yz$ on the sphere $x^2 + y^2 + z^2 = 8$ and the points at which they are attained.

Solution (as for white): The maximum value is 8, attained at $(\pm\sqrt{8},0,0)$, and the minimum value is -4, attained at $(0,\pm 2,\mp 2)$.

Exam 3 – LA3 (Yellow)

$f = 3xy - x^2y - xy^2 - 3$

$f_x = 3y - 2xy - y^2$ $\quad f_y = 3x - x^2 - 2xy$

$f_{xx} = -2y$ $\quad f_{xy} = 3 - 2x - 2y$ $\quad f_{yy} = -2x$

Cr. Pts:

$3y - 2xy - y^2 = 0$ $\quad 3x - x^2 - 2xy = 0$

$y(3 - 2x - y) = 0$

$y = 0$ $\quad y = 3 - 2x$

$3x - x^2 = 0$

$x = 0, x = 3$

$3x - x^2 - 2x(3 - 2x) = 0$

$-3x + 3x^2 = 0$

$x = 0$ $\quad x = 1$

$(y = 3)$ $\quad (y = 1)$

Thus at $(0,0)$ $\quad f_{xx}f_{yy} - f_{xy}^2 < 0$ $\quad$ Saddle

$(3,0)$ $\quad f_{xx}f_{yy} - f_{xy}^2 < 0$ $\quad$ Saddle

$(0,3)$ $\quad f_{xx}f_{yy} - f_{xy}^2 < 0$ $\quad$ Saddle

$(1,1)$ $\quad f_{xx}f_{yy} - f_{xy}^2 > 0, f_{xx} < 0$ $\quad$ Max.