Exam2 - MC (Pink)

1.
$$3x^2 + 3z^2\frac{\partial z}{\partial x} + 6yz^2 + 6xy\cdot 2z\frac{\partial z}{\partial x} = 0$$

$$\therefore\ 3 + 3\frac{\partial z}{\partial x} + 6 + 12\frac{\partial z}{\partial x} = 0 \qquad \therefore\ z = \frac{-9}{15} = -\frac{3}{5} \qquad \boxed{(c)}$$

2.
$$\Delta f \cong df = \frac{1}{2}\frac{1}{\sqrt{21-x^2-y^2}}(-2x)\,dx + \frac{1}{2}\frac{1}{\sqrt{21-x^2-y^2}}(-2y)\,dy$$

$$\therefore\ f(1.98, 1.04) = 4 - \frac{2}{4}(-.02) + \frac{1}{\cancel{2}}\frac{1}{4}(-\cancel{2}\cdot 1)(.04$$

$$= 4 + \frac{.04}{4} + \frac{1}{4}(-.04) = 4 \qquad \boxed{c}$$

3
$$\nabla f = (1 + e^y\cos(xz)z)\vec{i} + (1 + e^y\sin(xz))\vec{j} + (1 + e^y\cos(xz)x)\vec{k}\Big|_{(1,2,0)}$$
$$= \vec{i} + \vec{j} + (1+e^2)\vec{k}$$

$$\therefore\ \text{direct. der.} = (\vec{i} + \vec{j} + (1+e^2)\vec{k})\cdot\frac{(3\vec{i} - 2\vec{j} + c\vec{k})}{\sqrt{9+4+c^2}} = 0$$

$$\therefore\ 3 - 2 + c(1+e^2) = 0 \Rightarrow c = \frac{-1}{1+e^2} \qquad \boxed{b}$$

(4) $0 \le \dfrac{x^2}{x^2+y^2} \le 1$, $\sin(y) \to 0$ as $(x,y)\to 0$ $\therefore$ by Squeeze Theorem, $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2\sin(y)}{x^2+y^2} = 0$ $\boxed{a}$

LA1 ***Problem*** (pink). Find the maximum and minimum values of $f(x, y, z) = xz + y^2$ on the sphere $x^2 + y^2 + z^2 = 4$ and the points at which they are attained.

Solution (as for white): The maximum value is 4, attained at $(0, \pm 2, 0, 0)$, and the minimum value is -2, attained at $(\pm\sqrt{2}, 0, \mp\sqrt{2})$.

Exam2 - LA2 (PINK)

$f_x = 7y - 2xy - y^2$ $\qquad$ $f_y = 7x - 10 - x^2 - 2xy + 4y$

$f_{xx} = -2y$ $\qquad$ $f_{xy} = 7 - 2x - 2y$ $\qquad$ $f_{yy} = -2x + 4$

<u>Critical Pts</u>: $7y - 2xy - y^2 = 0$ $\qquad$ $7x - 10 - x^2 - 2xy + 4y = 0$

$y = 0$ $\qquad$ $y = 7 - 2x$

$7x - 10 - x^2 = 0$

$(x-5)(x-2) = 0$

$\therefore$ $x = 5$, $y = 0$

$x = 2$, $y = 0$

$7x - 10 - x^2 - 2x(7-2x) + 4(7-2x) = 0$

$3x^2 + 18 - 15x = 0$

$(x-3)(x-2) = 0$

$x = 3$ $\quad$ $y = 1$

$x = 2$ $\quad$ $y = 3$

<u>Finally</u>

at $(5,0)$ $\qquad$ $f_{xx} f_{yy} - f_{xy}^2 < 0$ $\qquad$ Saddle

$(2,0)$ $\qquad$ $f_{xx} f_{yy} - f_{xy}^2 < 0$ $\qquad$ Saddle

$(3,1)$ $\qquad$ $f_{xx} f_{yy} - f_{xy}^2 = \underbrace{(-2)(-2) - (-1)^2 > 0}_{<0} \rightarrow$ Max

$(2,3)$ $\qquad$ $f_{xx} f_{yy} - f_{xy}^2 < 0$ $\qquad$ Saddle

LA3 PINK

(a) Let $F(x,y,z) = \sqrt{x} + \sqrt{y} + \sqrt{3z}$. The normal vector to S at P:

$$\vec{N} = \nabla F(P) = \left\langle \frac{1}{2\sqrt{x_0}}, \frac{1}{2\sqrt{y_0}}, \frac{3}{2\sqrt{3z_0}} \right\rangle$$

The EQ. of the tangent plane to S at P:

$$\frac{1}{2\sqrt{x_0}}(x-x_0) + \frac{1}{2\sqrt{y_0}}(y-y_0) + \frac{3}{2\sqrt{3z_0}}(z-z_0) = 0$$

OR $$\frac{x}{2\sqrt{x_0}} + \frac{y}{2\sqrt{y_0}} + \frac{3z}{2\sqrt{3z_0}} = \underbrace{\frac{\sqrt{x_0}}{2} + \frac{\sqrt{y_0}}{2} + \frac{\sqrt{3z_0}}{2}}_{=\sqrt{C}}$$

(b)

$y = z = 0 \Rightarrow x_1 = 2\sqrt{x_0}\sqrt{C}$

$x = z = 0 \Rightarrow y_1 = 2\sqrt{y_0}\sqrt{C}$

$x = y = 0 \Rightarrow 3z_1 = 2\sqrt{3z_0}\sqrt{C}$

$$\Rightarrow x_1 + y_1 + 3z_1 = 2\sqrt{x_0}\sqrt{C} + 2\sqrt{y_0}\sqrt{C} + 2\sqrt{3z_0}\sqrt{C}$$
$$= \sqrt{4C}\sqrt{4C} = 4C,$$