Name: ___________________________
ID#: ___________________________

Midterm Exam

(11:00 am – 12:20 pm on March 1, 2018)

Problem 1. [10] Is the function
\[ f(z) = \ln(z + 1) - \ln(z - 1) \] (1)
multivalued? If it is, choose a branch with an appropriate branch cut.

Problem 2. [10] Solve the following equation using Laplace transform
\[ y(t) = 3t^2 - e^{-t} - \int_0^t y(\tau) e^{t-\tau} d\tau. \] (2)

Problem 3. [20] The idea is to find asymptotic solution of
\[ \frac{\partial c}{\partial t} + c \frac{\partial c}{\partial x} = \nu \frac{\partial^2 c}{\partial x^2}, \quad x \in \mathbb{R}, \] (3a)
\[ c(0, x) = F(x), \] (3b)
for \( \nu \to 0. \)

(a) [8] Using a sequence of transformations, first \( c = \partial \psi / \partial x \) and then \( \psi = -2\nu \ln \phi, \) reduce (3a) to the linear diffusion equation
\[ \frac{\partial \phi}{\partial t} = \nu \frac{\partial^2 \phi}{\partial x^2} \] (4)
and solve it with the initial condition corresponding to (3b).

(b) [12] Converting the solution for \( \phi(t, x) \) to that for \( c(t, x), \) determine its asymptotics for \( \nu \to 0 \) using Laplace’s method.

Problem 4. [20] Consider the function \( f(z) \) holomorphic in a neighborhood of a closure of the unit disk \( B(0). \)

(a) [10] Show that
\[ \overline{f}(0) = \frac{1}{2\pi i} \int_{\partial B} \frac{\overline{f}(\zeta)}{\zeta - z} d\zeta \] for all \( z \in B. \) (5)

(b) [10] Using the above result, prove that
\[ f(z) = \frac{1}{2\pi i} \int_{\partial B} \frac{\text{Re} f(\zeta) \zeta + z}{\zeta} d\zeta + i \text{Im} f(0) \] for all \( z \in B. \) (6)

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1The maximum score for the midterm is 40. If you get more than 40, the (positive) excess points will be added to the final exam. The final exam score will also have a maximum – any excess points will be lost.