Homework 4 / Take-home final exam

(due at 11:00 am on June 2, 2010)

Problem 1. [**12**] Why cannot planes fly in a heavy rain? Consider the "heavy rain" as consisting of drops of diameter 2 mm and mass concentration $2\,\mathrm{g/m^3}$. Estimate a relative increase in drag compared to the "no rain" situation.

Problem 2. [**5**] A liquid film of surface tension σ is stretched between two circular loops of radius a as shown. Find the equation(s) for $r(z)$. For which ratio d/a is the configuration indicated in figure 1 stable?

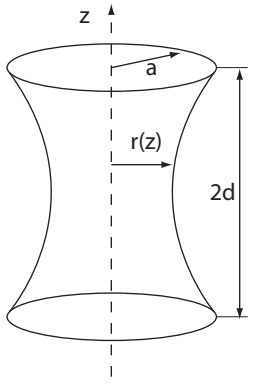


Figure 1: Soap film stretched between two circular loops.

Problem 3. [**8**] In which case – a drop on a strand of hair of radius b, cf. figure 2, or a free drop of the same volume – the pressure between the inside and outside (sometimes called "overpressure") is larger? Assume no gravity and the liquid perfectly wetting the hair, i.e. the contact angle at which the liquid meets the hair is zero.

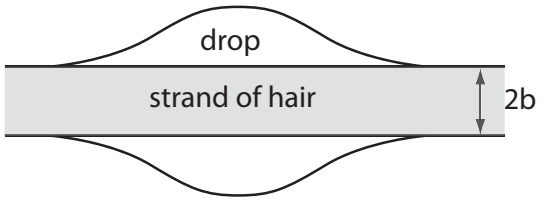
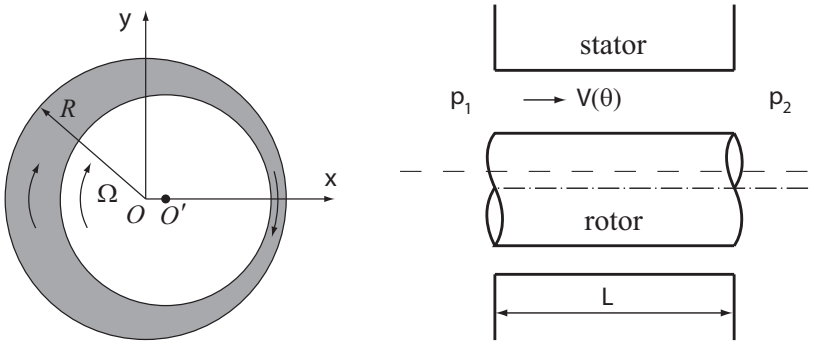


Figure 2: Drop on a strand of hair.

Problem 4. [**5**] Is it possible to have surface tension gradients in a flow of an inviscid fluid? Justify your answer.

Problem 5. [**15**] Discuss and quantify all possible hydrodynamic destabilizing effects in the bearing of finite length L shown in figure 3. The inner cylinder (rotor) is not fixed at the axis of symmetry of the stator and may move away from it under the action of destabilizing forces. Take typical values of the angular velocity $\Omega \sim 10^3\,\mathrm{rad/s}$, axial velocity $V \sim 0.5\,\mathrm{m/s}$, inner radius $R_1 \sim 4\,\mathrm{mm}$, outer radius $R_2 \sim 5\,\mathrm{mm}$, fluid kinematic viscosity $\nu \sim 5\,\mathrm{cSt}$.



(a) Cross-sectional view.

(b) Side view; θ is the angle in the cylindrical system of coordinates.

Figure 3: On finite-size bearing.