

### Homework 3

(due at 11:00 am on May 17, 2010)

**Problem 1.** Using the method of matched asymptotic expansion, solve

$$\epsilon \ddot{y} - \dot{y} + y = 0, \text{ with } y(0) = 0, y(1) = 1, \epsilon \ll 1.$$

Compare with the exact solution.

**Problem 2.** Develop a 2D version of the Kolmogorov-Obukhov theory of fully developed turbulence.

**Problem 3.** Formulate and solve the problem of viscous diffusion of a vortex line (i.e. a line where vorticity is concentrated at the time  $t = 0$ ).

**Problem 4.** The free surface of a liquid is one of constant pressure. If an incompressible fluid is placed in a cylindrical vessel and the whole is rotated with constant angular velocity  $\omega$ , show that the free surface becomes a paraboloid of revolution.

**Problem 5.** Determine a scaling for the period of oscillations of a gas bubble due to a deep explosion under water.

**Problem 6.** Find the pressure distribution in the slipper bearing assuming that the fluid motion is in the Stokes regime (cf. figure 1).

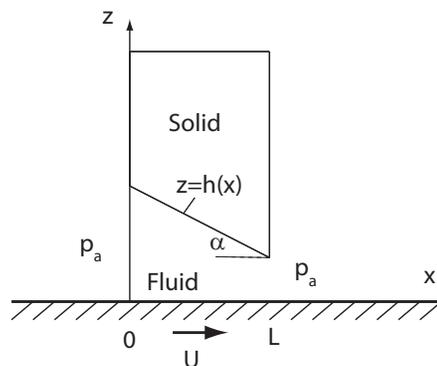


Figure 1: Slipper bearing. The plate moves with the constant velocity  $U$ . The lower boundary of the bearing, located at  $z = h(x)$ , is static and tilted at small angle  $\alpha$ .  $p_a$  is the ambient pressure.