Homework 3

(due at 3:30 pm on November 9, 2010)

Problem 1. Using the method of matched asymptotic expansion, solve

$$\epsilon\ddot{y} - \dot{y} + y = 0, \text{ with } y(0) = 0,\ y(1) = 1,\ \epsilon \ll 1.$$

Compare with the exact solution.

Problem 2. Develop a 2D version of the Kolmogorov-Obukhov theory of fully developed turbulence.

Problem 3. Formulate and solve the problem of viscous diffusion of a vortex line (i.e. a line where vorticity is concentrated at the time $t = 0$).

Problem 4. The free surface of a liquid is one of constant pressure. If an incompressible fluid is placed in a cylindrical vessel and the whole is rotated with constant angular velocity ω, show that the free surface becomes a paraboloid of revolution.

Problem 5. Determine a scaling for the period of oscillations of a gas bubble produced by a deep explosion under water.