A manifold has been machined so that when connected to a pressurized water supply n number of water jets are directed vertically upward as shown in figure 1 (note the direction of **g**). Each identical water jet has a cross sectional area, S_1 , and velocity, V_1 , at the exit of the manifold. A block with a flat bottom and mass, M, is placed over the jets and, when released, reaches some equilibrium height, h. After impact with the underside of the block, the water is deflected 90° and leaves the block as a thin sheet, the thickness of which is not essential for this problem. No additional forces support the block. Assume that the block is perfectly leveled when supported by the jets. Please do the following:

- (A) Develop an expression for the velocity of a water jet just prior to striking the underside of the block, V_2 . Clearly state all of the assumptions you use to arrive at your expression (you may want to add to this list of assumptions when completing the remainder of the problem).
- (B) Develop an expression for the area of a single water jet, S_2 , just prior to striking the underside of the block.
- (C) Develop an expression for the equilibrium height, h, of the block as a function of M, g, ρ , V_1 , and S_1 . In order to receive full credit you must indicate in your solution the control volume you are using (making it clear where the control surfaces are with respect to the block and manifold). Your control volume must also be appropriately labeled.



Figure 1: Block supported by water jets.

Please answer the following questions related to soap films and their retraction.

- (A) The units for surface tension are generally given as N/m, for example $\sigma_{H_2O} = 72 \times 10^{-3} N/m$. Show, using unit conversions, that these units are equivalent to J/m^2 (i.e. energy per unit area). Also include a brief statement explaining the physical meaning of surface tension.
- (B) Consider a soap film suspended on a circular frame. The diameter of the frame is D and the soap film has a uniform thickness h. The soap solution has a density, ρ , and surface tension, σ . Suppose that at some instant in time the soap film is uniformly released along the entire perimeter of the frame, so that you observe (if you had a high speed camera!) the edge of the soap film retracting with a velocity, U_{tc} . If you were interested in the order of magnitude of this velocity you could calculate it by considering that the surface energy of the film is converted into kinetic energy. Develop an expression, using the energy balance, to estimate the retraction velocity of the soap film, U_{tc} , as a function of the variables given.



Figure 2: Soap film on a circular frame before and after detachment (shown in perspective).

(C) Let us assume for the time being that we can create a soap film of any thickness we want, and we do so under normal atmospheric conditions (i.e. $T = 25 \,^{\circ}C$, $P = 101.3 \, kPa$). How thin would the soap film have to be such that the heat released due to retraction will lead to the complete evaporation of the soap film? First arrive at an expression for the film thickness in variable form and then substitute values to compute the thickness. Clearly state any assumptions that you use in your analysis.

Film Diameter	Surface Tension	Density	Specific Heat	Latent Heat of Vaporization
D	σ	ρ	C_p	L_{vap}
[cm]	[N/m]	$[kg/m^3]$	$[kJ/kg\cdot K]$	[kJ/kg]
5	0.035	1000	4.2	2260

Table 1: Information that you might find useful in your pursuit of excellence

(D) Briefly describe, in words, the flow of energy that leads to the complete evaporation of the soap film as analyzed in (C).

A plug of mass M is placed into a tube with cross sectional area A. The plug is held in place toward the closed end of the tube with a pin, and the space behind the plug, of volume V_O , is filled with a gas at some pressure P_O and temperature T_O . The side of the plug toward the open end of the tube is at atmospheric pressure. When the pin is released, there is no constraint holding the plug in place so it begins to move as the gas behind the plug expands. We will consider the case where there is no friction between the plug and the tube, there is no leakage of gas past the plug, there is always atmospheric pressure ahead of the plug, and the expansion of the gas behind the plug is isothermal. In order for the plug to achieve maximum velocity, the tube must be of optimal length in which case it is long enough for the gas to expand to atmospheric pressure just as the plug exits the tube.



Figure 3: (a) plug prior to release of pin, (b) after the pin has been released the plug has traveled down the tube.

Please do the following:

- (A) Draw and label a Free Body Diagram of the plug after the pin has been released and write down Newton's Second Law making sure to include all relevant forces acting on the plug.
- (B) Derive a differential equation for the velocity of the plug, U, as a function of distance traveled down the tube.
- (C) Using your differential equation derived in (B), or by some other appropriate means, determine the optimal length of the tube, X_{opt} , needed for the plug to achieve maximum velocity.
- (D) Solve your differential equation from (B) to find the maximum velocity of the plug, U_{max} , if the tube is of the optimal length X_{opt} .

Figure 4 shows a symmetric airfoil inclined at a moderate angle of attack. Well upstream of the airfoil, the flow field is uniform with velocity U and pressure P. Using you knowledge of fluid mechanics acquired this quarter, explain how the airfoil generates lift (a net force perpendicular to the direction of U). Begin by carefully drawing the flow field around the airfoil (draw streamlines). You are not required to do any calculations, but in order for your explanation to be adequate you will need to envoke concepts learned this quarter and these concepts need to be consistent with the flow field that you have sketched.



Figure 4: Symmetric airfoil at moderate angle of attack.