Midterm Test

(9:30-10:45 am on October 22, 2009)

Problem 1 (35). Water flowing from the oscillating slit shown in Fig. 1 produces a velocity field given by $\mathbf{v} = u_0 \sin[\omega(t - y/v_0)]\,\mathbf{i} + v_0\mathbf{j}$, where u_0, v_0, and ω are constants. Thus, the y component of velocity remains constant $v = v_0$ and the x component of velocity at $y = 0$ coincides with the velocity of the oscillating sprinkler head, $u_0 \sin[\omega t]$.

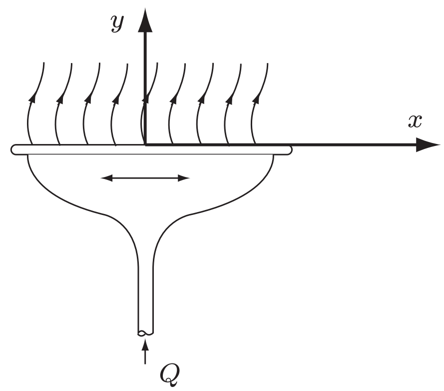

Figure 1: Oscillating sprinkler head.

(a:10) Determine the streamlines that pass through the origin at $t = 0$ and at $t = \pi/2\omega$.

(b:10) Determine the pathlines of the particle that were at the origin at $t = 0$ and at $t = \pi/2\omega$.

(c:15) Discuss the shape of the streakline that passes through the origin.

Problem 2 (25). An oil film of (dynamic) viscosity μ and thickness $h \ll R$ lies between a solid wall and a circular disk, as in Fig. 2. The disk is rotated steadily at angular velocity Ω. Noting that velocity and thus shear stress vary with radius r, derive a formula for the torque T required to rotate the disk. Neglect air drag, edge effects, and possible turbulence in the film; assume linear laminar velocity profile.

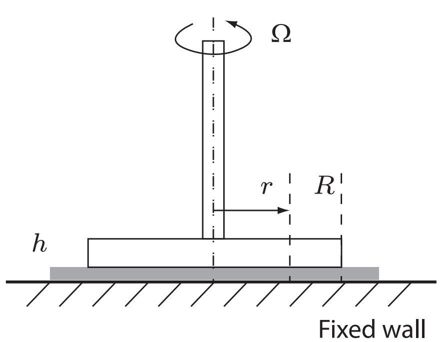

Figure 2: Rotating disk.

Problem 3 (25). A 0.6 mm diameter glass tube is inserted into water at 20° C as shown in Fig. 3(a). The surface tension of water at 20° C is $0.073\,N/m$. The contact angle of water with glass is approximately 0°. Take the density of water to be $10^3\,kg/m^3$. The air pressure can be considered constant.

(a:10) Determine the capillary rise of water in the tube.

(b:15) Can power be generated by drilling a hole in the tube just below the water level and feeding the water spilling out of tube just below the water level, as in Fig. 3(b)? Provide a physical explanation.

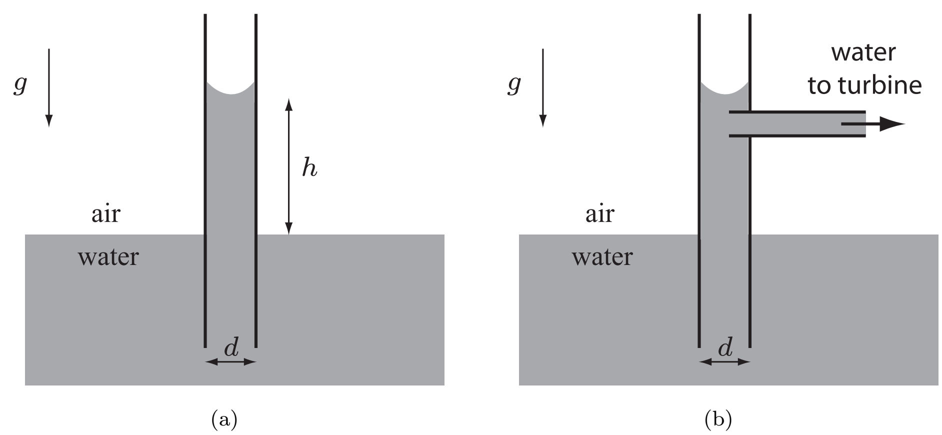

Figure 3: Capillary rise.