Homework 4
(due at 11:00 am on March 25, 2014)

Problem 1. Using the Laplace transform find the temperature $T$ distribution in a rod of unit length, one end of which is held at $T = 0$ and the other at $T^*$. The initial temperature distribution is $T = 0$ throughout the rod.

Problem 2. Applying the Laplace transform methods, solve the heat conduction problem on a half-line:

\[
\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, \quad x > 0, \quad t > 0,
\]
\[
u(0, t) = 0 \ (t > 0),
\]
\[
u(x, 0) = 1 \ (x > 0).
\]

Compare with the solution obtained by independent means, e.g. separation of variables.

Problem 3. Applying the Laplace transform methods, solve the heat conduction problem on a half-line:

\[
\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, \quad x > 0, \quad t > 0,
\]
\[
u(0, t) = u_0 \ cos \ \omega t \ (t > 0),
\]
\[
u(x, 0) = 0 \ (x > 0),
\]

and the solution should be bounded at $x = +\infty$.

Problem 4. Find a solution of the Laplace equation

\[
\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, \quad x > 0, \quad t > 0,
\]
\[
u(x, 0) = 0(x > 0),
\]
\[
\frac{\partial u}{\partial x}(0, y) = \begin{cases} 
- q \ (0 < y < b), \\
0 \ (y > b),
\end{cases}
\]

where $q = \text{const}$. Moreover, find the magnitude of the flow $q(x, 0) = \partial u/\partial y(x, 0)$ through the horizontal boundary of the domain.
Problem 5. Applying the Laplace transform methods, solve the following problem on a half-line:

\[
\frac{\partial^4 u}{\partial x^4} + \frac{\partial^2 u}{\partial t^2} = 0, \quad x > 0, \quad t > 0,
\]
\[
u(0, t) = u_0, \quad u_{xx}(t, 0) = 0 \; (t > 0),
\]
\[
u(x, 0) = u_t(x, 0) = 0 \; (x > 0),
\]

and the solution should be bounded at \( x = +\infty \).

Problem 6. Applying the Laplace transform methods, solve the following problem

\[
\frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial t^2} = 0, \quad l > x > 0, \quad t > 0,
\]
\[
u(0, t) = 0, \quad u_x(t, l) = 0 \; (t > 0),
\]
\[
u(x, 0) = 0, \quad u_t(x, 0) = -u_0 \; (x > 0).
\]