Homework 4

(due at 11:00 am on March 25, 2014)

Problem 1. Using the Laplace transform find the temperature T distribution in a rod of unit length, one end of which is held at $T = 0$ and the other at T^*. The initial temperature distribution is $T = 0$ throughout the rod.

Problem 2. Applying the Laplace transform methods, solve the heat conduction problem on a half-line:

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, \; x > 0, \, t > 0,$$
$$u(0, t) = 0 \; (t > 0),$$
$$u(x, 0) = 1 \; (x > 0).$$

Compare with the solution obtained by independent means, e.g. separation of variables.

Problem 3. Applying the Laplace transform methods, solve the heat conduction problem on a half-line:

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, \; x > 0, \, t > 0,$$
$$u(0, t) = u_0 \cos \omega t \; (t > 0),$$
$$u(x, 0) = 0 \; (x > 0),$$

and the solution should be bounded at $x = +\infty$.

Problem 4. Find a solution of the Laplace equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, \; x > 0, \, t > 0,$$
$$u(x, 0) = 0 (x > 0),$$
$$\frac{\partial u}{\partial x}(0, y) = \begin{cases} -q \; (0 < y < b), \\ 0 \; (y > b), \end{cases}$$

where $q =$ const. Moreover, find the magnitude of the flow $q(x, 0) = \partial u / \partial y (x, 0)$ through the horizontal boundary of the domain.

Problem 5. Applying the Laplace transform methods, solve the following problem on a half-line:

$$\frac{\partial^4 u}{\partial x^4} + \frac{\partial^2 u}{\partial t^2} = 0, \;\; x > 0, \, t > 0,$$
$$u(0,t) = u_0, \;\; u_{xx}(t,0) = 0 \;(t > 0),$$
$$u(x,0) = u_t(x,0) = 0 \;(x > 0),$$

and the solution should be bounded at $x = +\infty$.

Problem 6. Applying the Laplace transform methods, solve the following problem

$$\frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial t^2} = 0, \;\; l > x > 0, \, t > 0,$$
$$u(0,t) = 0, \;\; u_x(t,l) = 0 \;(t > 0),$$
$$u(x,0) = 0, u_t(x,0) = -u_0 \;(x > 0).$$

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