Homework 2

(due at 11:00 am on February 11, 2014)

Problem 1. Using separation of variables, solve the problem of oscillations of a circular membrane of radius r_0:

$$\begin{aligned} u_{tt} - a^2 \Delta u &= 0,\ r < r_0, a \in \mathbb{R}, \\ u(r,\theta,0) &= f_1(r,\theta),\ u_t(r,\theta,0) = f_2(r,\theta), \\ u(r_0,\theta,t) &= 0,\ t > 0. \end{aligned}$$

Here (r,θ) are polar coordinates. Also, prove that $\Delta u = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2}$.

Problem 2. Based on the understanding of the D'Alembert solution and the method of characteristics, solve the following problem for semi-infinite string:

$$\begin{aligned} u_{tt} - a^2 u_{xx} &= 0,\ x > 0,\ t > 0, a \in \mathbb{R}, \\ u(x,0) &= \phi(x),\ u_t(x,0) = \psi(x),\ x > 0, \\ u(0,t) &= 0,\ t > 0. \end{aligned}$$

Problem 3. Solve

$$\begin{aligned} u_{tt} - a^2 u_{xx} &= 0,\ x > 0,\ t > 0, a \in \mathbb{R}, \\ u(x,0) &= \phi(x),\ u_t(x,0) = \psi(x),\ x > 0, \\ u(0,t) &= \mu(t),\ t > 0. \end{aligned}$$

Problem 4. Use the Kirchhoff formula for the solution of the IVP of 3D wave equation to predict the pressure at any point and any time after the explosion of a balloon of radius r_b and initial pressure p_b at the time instant $t = 0$. Note: the pressure obeys the 3D wave equation.

Problem 5. Use the Hadamard method of descent to deduce the solution of the IVP for the wave equation in $\mathbb{R}^2$ from the Kirchhoff formula for the solution of the IVP of 3D wave equation.

Problem 6. Solve the boundary/initial value problem on a half-line:

$$\begin{aligned}
\frac{\partial u}{\partial t} &= k\frac{\partial^2 u}{\partial x^2} + f(x,t),\ x \in \mathbb{R}^+,\ t > 0,\\
u(x,0) &= 0;\ x \in \mathbb{R}^+,\\
u(0,t) &= 0,\ t \in [0,T].
\end{aligned}$$

Problem 7. Solve the boundary/initial value problem

$$\begin{aligned}
\frac{\partial u}{\partial t} &= k\frac{\partial^2 u}{\partial x^2},\ 0 < x < L,\ t > 0,\\
u(0,t) &= A,\ u(L,t) = B,\ t > 0,\\
u(x,0) &= f(x),\ 0 < x < L,
\end{aligned}$$

where A and B are constants.

Problem 8. Solve the boundary/initial value problem

$$\begin{aligned}
\frac{\partial u}{\partial t} &= k\frac{\partial^2 u}{\partial x^2} + Ae^{-t/\tau},\ 0 < x < L,\ t > 0,\\
\left.\frac{\partial u}{\partial x}\right|_{x=0} &= 0;\ \left.\frac{\partial u}{\partial x}\right|_{x=L} = 0,\ t > 0,\\
u(x,0) &= 0,\ 0 < x < L,
\end{aligned}$$

where A is a constant.