Homework 1

(due at 11:00 am on January 28, 2014)

Problem 1. Integrate the following Cauchy problem for the first-order equation

$$u_t + uu_x + u = 0,$$
$$u(x,0) = f(x).$$

Problem 2. Obtain the complete integral of the following equation

$$u_x^2 + yu_y = u.$$

Problem 3. Determine the time when the wave – the solution of the following equation – breaks in the usual sense of wave breaking,

$$u_t + C(u)u_x = 0,$$
$$u(x,0) = f(x).$$

Problem 4. Find where the following equation is elliptic, hyperbolic and parabolic:

$$(l+x)u_{xx} + 2xyu_{xy} - y^2u_{yy} = 0,\ l \in \mathbb{R}.$$

Problem 5. Reduce to the canonical form

$$(a)\ u_{xx} + yu_{yy} + \frac{1}{2}u_y = 0;$$
$$(b)\ u_{xx}\sin^2 x - 2yu_{xy}\sin x + y^2u_{yy} = 0.$$

Problem 6. Construct the Green's function for the Laplace equation in a half-space $(-\infty < x, y, < \infty,\ z \geq 0)$. *Hint*: use the method of reflections.

Problem 7. Construct the Green's function for the Laplace equation in a three-dimensional ball of radius a. *Hint*: use separation of variables.

Problem 8. Give a physical interpretation of the maximum principle for the Laplace equation, say in terms of temperature.

Problem 90. Find the volume ("domain") potential of a ball of radius a with a constant charge density ρ_0.

Problem 10. Find the single layer potential of a sphere of radius a with a constant charge density ν_0.

Problem 11. Find the double layer potential of an interval $[-a, a]$ with a constant dipole moment density.