Math 371: Assignment 9

Due Thursday April 2, before class.

1. Deer can be found in three habitats: water (w), grass (g) and sleeping areas (s). The transition matrix associated with movement between these areas is

$$P = \begin{bmatrix} 0.6 & 0.25 & 0.25 \\ 0.2 & 0.5 & 0.25 \\ 0.2 & 0.25 & 0.5 \end{bmatrix}$$

   so that the dynamics of the Markov process are given by

$$\mathbf{u}_{t+1} = P\mathbf{u}_t$$

   where the vector $\mathbf{u}_t = (w_t, g_t, s_t)^T$ describes the probabilities of an individual being found in each of the three habitat types at time step t.

   Draw the directed graph associated with this Markov process. Put weight on the edges to denote the probabilities in the matrix.

   If at time t_0 the deer is equally likely to found in all places, in which place is the deer more likely to be found after one time step, after two time steps? What is the fixed probability distribution $\mathbf{u}^*$ which is found after many time steps? Recall that this is the eigenvector of the system associated with the eigenvalue 1.

2. If an individual is subject to a constant death rate, then the probability of individual surviving to time t, given by $q(t)$, satisfies the following differential equation

$$\frac{dq}{dt} = -\mu q \qquad q(0) = 1.$$

   Write down the solution to this differential equation. Use this solution and the attached figure to estimate the lapwing death rate μ. You will want to look at the scales for the axes carefully. HINT: at time $t = 4.5$ years there are 100 lapwings remaining. At time $t = 9$ years there are 10 lapwings remaining. IE, over 4.5 years the probability of a single individual surviving is 0.1. Use this observation and the formula for $q(t)$ in deriving the value of the parameter μ.

   Let $p(t)$ be the probability that the lapwing is dead by time t. Write down a formula for $p(t)$. (Recall, $p(t) + q(t) = 1$.) Given an individual lapwing, after how many years is there a 50 per cent probability that it has died, a 95 per cent probability that it has died?

   Assuming that the lapwings die independently from one another, the probability that a cohort of n_0 lapwings are all dead by time t is $(p(t))^{n_0}$ (why?). In other words, if T is the time to extinction for the population of n_0 individuals, then the probability that all individuals are dead by time t can be written as

$$\Pr(T \leq t) = (p(t))^{n_0}.$$

Given a cohort of 100 lapwings, what is the probability that there are individuals remaining after 10 years, 20 years, and 30 years?

3. Exercise 5.8.8, text .

4. Matlab exercises 8.3.4 and 8.3.6. Please hand in both the Matlab code used to answer the exercises, and output from the code, for this question only. Clearly mark the relevant question number and section on both the code and the output. The Matlab Course is given at http://www.math.ualberta.ca/ mlewis/Courses/371.html

Figure 2.5. Composite age-specific survivorship curve for the lapwing, *Vanellus vanellus*, based on ringed birds found dead in Europe. Data are presented from the end of the first year of life onward. (Reproduced from Hutchinson, 1978, by permission of Yale University Press.)